Introduction to Nuclear Engineering

Third Edition

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The author and publisher of this book have used their best efforts in preparing this book. These efforts include the development, research, and testing of the theories and programs to determine their effectiveness.

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Heat Removal from Nuclear Reactors

For a reactor to operate in a steady state, with an internal temperature distribution that is independent of time, all of the heat released in the system must be removed as fast as it is produced. This is accomplished, in all reactors except those operating at very low power levels, by passing a liquid or gaseous coolant through the core and other regions where heat is generated. The nature and operation of this coolant system is one of the most important considerations in the design of a nuclear reactor.

The temperature in an operating reactor varies from point to point within the system. As a consequence, there is always one fuel rod, usually one of the rods near the center of the reactor, that at some point along its length is hotter than all the rest. This maximum fuel temperature is determined by the power level of the reactor, the design of the coolant system, and the nature of the fuel. However, metallurgical considerations place an upper limit on the temperature to which a fuel rod can safely be raised. Above this temperature, there is a danger that the fuel may melt, which can lead to the rupture of the cladding and release of fission products. One of the major objectives in the design of a reactor coolant system is to provide for the removal of the heat produced at the desired power level while ensuring that the maximum fuel temperature is always below this predetermined value.

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It should be noted that, from a strictly nuclear standpoint, there is no theoretical upper limit to the power level that can be attained by any critical reactor having sufficient excess reactivity to overcome its negative temperature coefficient. Thus, by removing control rods and placing a reactor on a positive period, its power could be increased indefinitely were it not for the fact that eventually a point would be reached where the coolant system is no longer able to remove all of the heat being produced. Beyond this point, the fuel would heat up and eventually a portion of the core would melt down. This situation is avoided in the actual operation of a reactor by reinserting some of the control rods or adding boron to the shim system to return the reactor to critical when a desired power level has been reached. Throughout the present chapter, it is assumed that the reactors under consideration are critical and operating at a constant power.

Before beginning the discussion of the design of reactor coolant systems, a word is in order concerning units. As pointed out in Appendix I, where the matter of units is considered in some detail, the SI system of units has been adopted by most of the nations of the world, with the exception of the United States. All U.S. scientists and many U.S. engineers and engineering societies also use this system. However, there is still strong resistance to the adoption of SI units. As a result, most American manufacturers of nuclear power equipment base their designs on the English system. Since this state of affairs is likely to persist for several years to come, English units are used throughout the present chapter as the primary unit. SI units are provided where appropriate. Tables for conversion from English to SI units are given in Appendix I.

8.1 GENERAL THERMODYNAMIC CONSIDERATIONS

From a thermodynamic point of view, a nuclear reactor is a device in which energy is produced and transferred to a moving fluid. Thus, as indicated in Fig. 8.1, heat is released in a reactor at the rate of q BTU/hr or watts and absorbed by the coolant, which enters the reactor at the temperature T_{in} and exits from the reactor at the temperature T_{out}, passing through the system at the rate of w lb/hr or kg/hr.

With all power reactors except the BWR and RBMK, there is no (net) change in phase of the coolant as it passes through the reactor; that is, the coolant does not boil. In these reactors, the heat from the reactor merely increases the temperature of the coolant, a process that occurs at essentially constant pressure—namely, the reactor coolant pressure. The heat in BTUs or joules required to raise the temperature of a unit mass of coolant from the temperature T_{in} to T_{out} is

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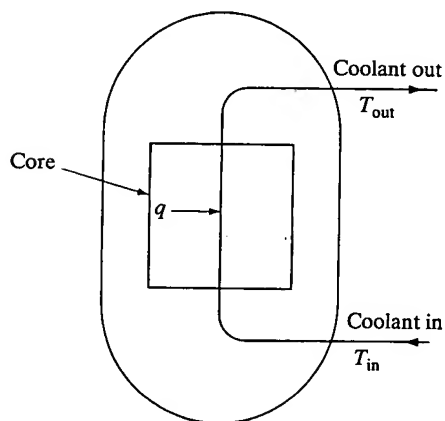emperature of
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Figure 8.1   Schematic drawing of coolant flow through a reactor.

where $c_p(T)$ is the specific heat at constant pressure per unit mass of the coolant. Since w lb or kg of coolant flows through the reactor per hour, the rate at which heat produced in the reactor is absorbed by the coolant is given by

$$q = w \int_{T_{in}}^{T_{out}} c_p(T)\,dT. \tag{8.1}$$

Equation (8.1) can also be written in terms of the thermodynamic function *enthalpy*. This is defined for a substance by the relation

$$h \equiv u + Pv, \tag{8.2}$$

where h is the enthalpy per unit mass of the substance (or *specific enthalpy*), u is its internal energy per unit mass, P is the pressure, and v is the *specific volume* of the substance (i.e., the volume per unit mass ft³/lb or m³/kg). The units of h are Btu/lb or joules/kg. In thermodynamics, it is shown that when heat is added to a substance at constant pressure, essentially all of the heat is used to increase its enthalpy.[1] Thus, if h_{in} and h_{out} are the specific enthalpies of the coolant entering and leaving the reactor, respectively, it follows that

$$h_{out} = h_{in} + \int_{T_{in}}^{T_{out}} c_p(T)\,dT \tag{8.3}$$

[handwritten] all reactors but BWRs and RBMK

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[1] A very small amount of heat may also be used to increase the kinetic energy of the substance or change its gravitational potential energy by virtue of raising the substance from a lower to a higher level in the earth's gravitational field. However, both of these contributions are negligible for reactor coolants.

It also follows that Eq. (8.1) can be written as

$$q = w(h_{out} - h_{in}). \tag{8.4}$$

The situation is somewhat more complicated in the case of a BWR or an RBMK. In these reactors, a portion of the water passing through the core is vaporized to steam, exits from the reactor via a steam pipe to the turbine, and later returns as feed water from the condenser and reheaters. Most of the water that goes through the core is recirculated within the reactor. However, in steady-state operation, there is no net absorption of heat by the recirculating water. Therefore, although the incoming feed water mixes with the recirculating water before passing through the core, the overall effect of the reactor is simply to vaporize the feed water, as is indicated schematically in Fig. 8.2.

The change in enthalpy from the point where the feed water enters a BWR to the point where the steam exits from the reactor consists of two stages. The water is first heated from its entering temperature T_{in} to the temperature at which it boils—that is, the *saturation temperature* T_{sat} for the given system pressure. The water temperature does not rise above this value. The associated increase in enthalpy is

$$h_f = h_{in} + \int_{T_{in}}^{T_{sat}} c_p(T)\,dT. \qquad \leftarrow \text{BWR}$$

This is the enthalpy of the saturated water. With the onset of boiling, the water absorbs an amount of heat equal to the *heat of vaporization*, denoted by h_{fg}, per unit mass of water that changes phase. Thus, the specific enthalpy of the steam is
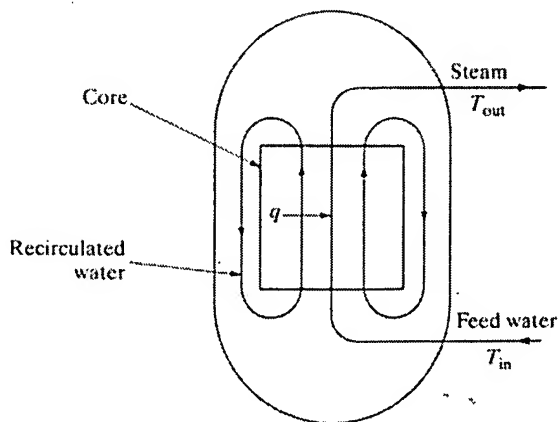

Figure 8.2 Schematic drawing of coolant flow through a BWR.

$$h_{out} = h_f + h_{fg} \tag{8.5}$$

$$= h_{in} + \int_{T_{in}}^{T_{sat}} c_P(T)dT + h_{fg}$$

Again, the rate at which heat is absorbed by the coolant is given by Eq. (8.4),

$$q = w(h_{out} - h_{in})$$

where w is the steam or feed water flow rate (the two are equal) in lb/hr or kg/hr.

The advantage of expressing q in terms of enthalpy is that h has been computed using Eq. (8.3) or Eq. (8.5) and is available in tabular form for a number of substances. Abridged tables of h (these are called *steam tables*) are given in Appendix IV for water, helium, and sodium. It should be noted that only the *difference* in enthalpy between two different temperatures is meaningful and useful; the absolute value of enthalpy has no meaning. Thus, the zero of enthalpy can be set arbitrarily, and the values of enthalpy computed from this starting point. In the case of water, for example, in the English system of units, h is taken to be zero for the saturated liquid at 32°F or in SI units as 0°C.

Again, for water, it is observed in Appendix IV that the specific enthalpy is tabulated for saturated water and steam as a function of the temperature, and associated pressure in Table IV.1, and as a function of pressure and the associated temperature in Table IV.2. (It must be remembered that there is only one pressure for a given temperature; conversely, there is one temperature for a given pressure at which water and steam are in equilibrium—i.e., at which water will boil.) However, the enthalpy of a liquid is not a sensitive function of pressure. Thus, the enthalpy of water in the pressurized water reactor is nearly equal to the value of h indicated in the tables for saturated liquid at the actual water temperature independent of its pressure.[2] This procedure is used in the first of the following two examples, which illustrate the use of Eq. (8.4) for PWR and BWR systems.

Example 8.1

A PWR operates at a thermal power of 3,025 MW. Water enters the reactor at 542.6°F and flows through the reactor at a rate of 136.3×10^6 lb/hr. The system pressure is 2,250 psia.[3] At what temperature does the coolant leave the reactor?

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[2]Data for correcting saturation values of h for higher pressure are given in Table 4 of J. H. Keenan and F. G. Keyes, *Thermodynamic Properties of Steam Including Data for the Liquid and Solid Phases*, New York: Wiley, 1959. In the temperature region from 400°F to 600°F and up to pressures of 300 psi, the correction for h amounts to less than 1%.

[3]Psia means *absolute-pressure* in lb/in³. The difference between the reactor pressure and atmospheric pressure is called *gauge pressure* and is abbreviated psig.

Solution. According to Table I.9, 1 kW = 3,412 Btu/hr. Then

$$q = 3,205 \times 10^3 \times 3,412 = 10,321 \times 10^6 \text{ Btu/hr.}$$

From Eq. (8.4), the rise in enthalpy through the core is

$$h_{out} - h_{in} = \frac{q}{w} = \frac{10,321 \times 10^6}{136.3 \times 10^6} = 75.7 \text{ Btu/lb.}$$

Interpolating from the values of enthalpy in Table IV.1 for saturated water at 540°F and 550°F gives the value of h at 542.6°F as 539.7 Btu/lb. The enthalpy of the emerging water is then

$$h_{out} = 539.7 + 75.7 = 615.4 \text{ Btu/lb.}$$

A second interpolation from Table IV.1 shows that this corresponds to water at a temperature of 599.1°F. [*Ans.*]

Example 8.2

A BWR operating at pressure of 6.895 MPa produces 2.93×10^6 kg of steam per hour. Feed water enters the reactor at 190.6°C. (a) What is the temperature of the steam? (b) At what power is the reactor operating?

Solution.

1. The steam emerges at essentially the temperature at which the water boiled. According to Table IV.2, at a pressure of 6.895 MPa, this is 284.86°C. [*Ans.*]
2. From Table IV.2, h of the steam is 2,773.2 kJ/kg, while from Table IV.1 at 190.6°C, h of the feed water is 807.8 kJ/kg. Then Eq. (8.4) gives

$$q = 2.93 \times 10^6(2,773.2 - 807.8) = 5,758 \times 10^6 \text{ J/hr} = 1,599 \text{ MW. [}Ans.\text{]}$$

8.2 HEAT GENERATION IN REACTORS

The starting point in the design of a reactor cooling system is the determination of the spatial distribution of the heat produced within the reactor. This problem is discussed in this section.

It was pointed out in Chapter 3 that the energy released in fission appears in several forms—as kinetic energy of the fission neutrons, as prompt fission γ-rays, as γ-rays and β-rays from the decay of fission products, and in the emission of neutrinos. With the exception of the neutrinos, virtually all of this energy is ultimately absorbed somewhere in the reactor. However, because these various radiations are attenuated in different ways by matter, their energy tends to be deposited in different locations. In the following discussion of the deposition of energy, it is assumed

that the reactor fuel is in the form of individual fuel rods. This is the case for virtually all power reactor concepts except the molten salt breeder reactor (see Section 4.5).

Heat Production in Fuel Elements (Fuel Rods)

As indicated in Table 3.6, the fission fragments have a kinetic energy of about 168 MeV per fission. These highly charged particles have an extremely short range (see Section 3.9), and therefore their energy is deposited near the site of the fission within the fuel. Similarly, most of the 8 MeV of the fission product β-rays is also deposited in the fuel. However, many of the γ-rays from the decaying fission products and those emitted directly in fission pass out of the fuel since they are less strongly attenuated than charged particles. Some of these γ-rays are absorbed in the surrounding coolant and/or moderator, in the thermal shield, or in the radiation shielding that surrounds the reactor. However, because of the proximity of fuel rods in most reactors, many of the γ-rays are intercepted by and absorbed in neighboring rods.

The prompt neutrons are emitted with a total kinetic energy of about 5 MeV per fission. In a thermal reactor, the bulk of this energy is deposited in the moderator as the neutrons slow down. The capture γ-rays emitted following the absorption of these neutrons in nonfission reactions are therefore produced and absorbed throughout the reactor. In a fast reactor, the fission neutrons slow down very little before they are absorbed, and their kinetic energy appears as an addition to the energy of the capture γ-rays. The delayed neutrons contribute negligibly to the energy of a reactor. As already noted, none of the energy of the neutrinos is retained within a reactor.

It should be clear from the foregoing remarks that the spatial deposition of fission energy depends on the details of the reactor's structure. Nevertheless, for preliminary calculations, it may be assumed that approximately one-third of the total γ-ray energy—about 5 MeV—is absorbed in the fuel. This, together with the 168 MeV from the fission fragments and 8 MeV from the β-rays, gives 181 MeV per fission (about 90% of the recoverable fission energy), which is deposited in the fuel, most in the immediate vicinity of the fission site. The remainder (about 20 MeV) of the recoverable energy is deposited in the coolant and/or moderator, in various structural materials, and in the blanket, reflector, and shield.

The rate at which fission occurs in the fuel, and hence the rate of production of heat, varies from fuel rod to fuel rod, and it is also a function of position within any given rod. In particular, if E_d is the energy deposited locally in the fuel per fission, then the rate of heat production per unit volume at the point $\mathbf{r}$ is given by

the expression[4]

$$q'''(\mathbf{r}) = E_d \int_0^\infty \Sigma_{fr}(E)\phi(\mathbf{r},E)\,dE, \tag{8.6}$$

where $\Sigma_{fr}(E)$ is the macroscopic fission cross-section of the fuel and $\phi(\mathbf{r}, E)$ is the energy-dependent flux as a function of position. The natural units of q''' are MeV/sec-cm³. However, for engineering calculations, q''' should be converted to units of Btu/hr-ft³ or kW/lit using the conversion factors in Appendix I.

In the thermal reactor, most of the fissions are induced by thermal neutrons. In this case, according to the results of Section 5.9, the integral in Eq. (8.6) can be written as

$$q'''(\mathbf{r}) = E_d \overline{\Sigma}_{fr}\phi_T(\mathbf{r}), \tag{8.7}$$

where $\overline{\Sigma}_{fr}$ is the thermal fission cross-section of the fuel and $\phi_T(\mathbf{r})$ is the thermal flux. In the framework of a multigroup calculation, Eq. (8.6) can also be expressed in the form

$$q'''(\mathbf{r}) = E_d \sum_g \Sigma_{fg}\phi_g(\mathbf{r}), \tag{8.8}$$

where the sum is carried out over all the groups and the notation is the same as in Section 5.8.

The spatial dependence of the flux depends on the geometry and structure of the reactor. However, many heat removal calculations are carried out for the theoretical case of the bare, finite cylinder. For a thermal reactor, the thermal flux is then (see Table 6.2)

$$\phi_T(r, z) = \frac{3.63P}{E_R \overline{\Sigma}_f V} J_0\left(\frac{2.405r}{\widetilde{R}}\right)\cos\left(\frac{\pi z}{\widetilde{H}}\right), \tag{8.9}$$

where P is the total power of the reactor in joules, E_R is the recoverable energy per fission in joules, V is the reactor volume in cm³, and $\widetilde{R}$ and $\widetilde{H}$ are its outer dimensions in centimeters to the extrapolated boundaries. In obtaining Eq. (8.9), it was assumed that the fuel is homogeneously distributed throughout the reactor.

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[4]The following notation for heat production and flow is well established in heat transfer literature and is used in this chapter:

q''' :   power density, rate of energy production per unit volume; BTU/hr-ft³ or kW/lit.

q'' :   heat flux; BTU/hr-ft² or W/cm²,

q' :   linear power; Btu/hr-ft or kW/m,

q :   heat transfer rate; Btu/hr or MW.

and $\overline{\Sigma}_f$ is the value of the macroscopic fission cross-section of this mixture in units of cm^{-1}.

Equation (8.9) can also be used to approximate the flux in a reactor where the fuel is contained in separate fuel rods, provided the value of $\overline{\Sigma}_f$ is computed for the equivalent homogeneous mixture. For instance, suppose that there are n fuel rods of radius a and length H, the height of the core. Then if $\overline{\Sigma}_{fr}$ is the macroscopic fission cross-section of the rod, the total fission cross-section in the entire core[5] is $\overline{\Sigma}_{fr} \times n\pi a^2 H$. Therefore, the average value of $\overline{\Sigma}_f$ in the core is

$$\overline{\Sigma}_f = \frac{\overline{\Sigma}_{fr} n \pi a^2 H}{\pi R^2 H} = \frac{\overline{\Sigma}_{fr} n a^2}{R^2} \tag{8.10}$$

and the flux is

$$\phi_T(r, z) = \frac{3.63 P R^2}{E_R \overline{\Sigma}_{fr} V a^2 n} J_0\left(\frac{2.405r}{\widetilde{R}}\right) \cos\left(\frac{\pi z}{\widetilde{H}}\right)$$

$$= \frac{1.16 P}{E_R \overline{\Sigma}_{fr} H a^2 n} J_0\left(\frac{2.405r}{\widetilde{R}}\right) \cos\left(\frac{\pi z}{\widetilde{H}}\right) \tag{8.11}$$

where $V = \pi R^2 H$ has been substituted.

When the expression for ϕ_T in Eq. (8.11) is introduced into Eq. (8.7), the rate of heat production per unit volume of a fuel rod becomes

$$q'''(r, z) = \frac{1.16 P E_d}{H a^2 n E_R} J_0\left(\frac{2.405r}{\widetilde{R}}\right) \cos\left(\frac{\pi z}{\widetilde{H}}\right) \tag{8.12}$$

It should be noted that, in this procedure for obtaining q''', any variation of the flux across the diameter of the fuel rods has been ignored. The dependence of q''' on r in Eq. (8.12) gives the change in the flux from rod to rod across the diameter of the core—not across any individual fuel rod. Indeed, given the derivation of Eq. (8.12), q''' must be taken to be constant across individual rods. This does not introduce significant errors in heat transfer calculations, especially for the small-diameter, weakly absorbing (low-enrichment) fuel rods currently used in most power reactors.

From Eq. (8.12), it is evident that the maximum rate of heat production occurs in the middle ($z = 0$) of the central rod ($R = 0$). In this case, both functions in Eq. (8.12) are unity, and the maximum value of q''' is

$$q'''_{max} = \frac{1.16 P E_d}{H a^2 n E_R} \tag{8.13}$$

[5]Recall that macroscopic cross-section is cross-section per unit volume.

The maximum rate of heat production in a noncentral rod located at $r \neq 0$ is

$$q_{max}^{m}(r) = q_{max}^{m} J\left(\frac{2.405r}{\widetilde{R}}\right) \tag{8.14}$$

The total rate at which heat is produced in any fuel rod is given by the integral

$$q_r(r) = \pi a^2 \int_{-H/2}^{H/2} q'''(r, z) dz.$$

Introducing $q'''(r, z)$ from Eq. (8.12) gives

$$q_r(r) = \frac{1.16\pi P E_d}{H n E_R} J_0\left(\frac{2.405r}{R}\right) \int_{-H/2}^{H/2} \cos\left(\frac{\pi z}{H}\right) dz$$

$$= \frac{2.32 P E_d}{n E_R} J_0\left(\frac{2.405r}{R}\right). \tag{8.15}$$

The formulas derived previously for the bare cylindrical reactor should not be taken too seriously for calculations of the heat production in a real reactor. In particular, Eq. (8.13) considerably overestimates the value of q_{max}^{m} for a reflected and/or nonuniformly fueled reactor, which has a smaller maximum-to-average flux ratio than a bare reactor (see Section 6.3). To see this effect, it is first noted that

$$q_{max}^{m} = \overline{\Sigma}_{fr} E_d \phi_{max}, \tag{8.16}$$

where ϕ_{max} is the maximum value of the thermal flux. The total reactor power is given by

$$P = \overline{\Sigma}_f E_R \phi_{av} V, \tag{8.17}$$

where ϕ_{av} is the average thermal flux and $\overline{\Sigma}_f$ is again the macroscopic fission cross-section averaged over the entire core volume V. Dividing Eq. (8.16) by Eq. (8.17) and rearranging gives

$$q_{max}^{m} = \frac{P \overline{\Sigma}_{fr} E_d \phi_{max}}{\overline{\Sigma}_f E_R \phi_{av} V} = \frac{P \overline{\Sigma}_{fr} E_d \Omega}{\overline{\Sigma}_f E_R V}. \tag{8.18}$$

where Ω is the maximum-to-average flux ratio. Finally, introducing Eq. (8.10) for $\overline{\Sigma}_f$ yields

$$q_{max}^{m} = \frac{P E_d R^2 \Omega}{a^2 n V E_R} = \frac{P E_d \Omega}{\pi H a^2 n E_R}. \tag{8.19}$$

Suppose that $\Omega = 2.4$, a reasonable value for an actual reactor. Then comparing Eqs. (8.13) and (8.19) shows that

$$(q_{max}''')_{actual} = \frac{\Omega}{1.16\pi}(q_{max}''')_{bare} \simeq \frac{2}{3}(q_{max}''')_{bare}.$$

Example 8.3

The extrapolated dimensions of a certain pressurized water reactor are $\widetilde{R} = 67$ in and $\widetilde{H} = 144$ in. The reactor operates at the thermal power of 1,893 MW. It contains 193 fuel assemblies, each consisting of 204 UO_2 fuel rods 0.42 in. in diameter. Assuming that the assemblies are uniformly distributed throughout the reactor, calculate the total energy production rate and the maximum energy production rate per ft^3 in rods located (a) at the axis of the reactor, (b) 20 in from the axis.

Solution.

1. There are $n = 193 \times 204 = 39,372$ rods in the reactor. Assuming that $E_d = 180$ MeV and $E_R = 200$ MeV, Eq. (8.15) gives for the rod at $r = 0$

$$q_r(0) = \frac{2.32 \times 1893 \times 180}{39,372 \times 200} = 0.100 \text{MW}$$

$$= 3.43 \times 10^5 \text{ Btu/hr. } [Ans.]$$

Comparing Eqs. (8.13), (8.14), and (8.15) shows that

$$q_{max}''' = \frac{1}{2Ha^2}q_r(0).$$

Therefore,

$$q_{max}''' = \frac{3.43 \times 10^5}{2 \times \left(\frac{144}{12}\right) \times \left(\frac{0.21}{12}\right)^2}$$

$$= 4.66 \times 10^7 \text{ Btu/hr-ft}^3 \text{ (482.3 kW/lit). } [Ans.]$$

2. To obtain similar results for a rod located at $r = 20$ in, it is merely necessary to multiply the prior answers by the factor appearing in Eqs. (8.14) and (8.15), yielding,

$$J_0\left(\frac{2.405r}{\widetilde{R}}\right) = J_0\left(\frac{2.405 \times 20}{67}\right) = J_0(0.718) = 0.875.$$

Thus, for this rod,

$$q_r(20 \text{ in}) = 3.43 \times 10^5 \times 0.875$$

$$= 3.00 \times 10^5 \text{ Btu/hr (87.9 kW) } [Ans.]$$

and

$$q_{max}'''(20 \text{ in}) = 4.66 \times 10^7 \times 0.875$$
$$= 4.08 \times 10^5 \text{ Btu/hr-ft}^3 (4.2 \text{ kW/lit}). \quad [Ans.]$$

Radiation Heating

As noted already, roughly 10% of the recoverable energy of fission is absorbed outside the fuel. In thermal reactors, the kinetic energy of the fission neutrons is deposited in the surrounding moderator and coolant in more or less the same spatial distribution as the fissions from which these neutrons originate. However, only 2% to 3% of the fission energy appears in this form, and it is often assumed that this kinetic energy is deposited uniformly throughout the core.

The calculation of the energy deposition from the longer range γ-rays is a more difficult problem. In principle, this can be determined by evaluating the integral

$$q'''(\mathbf{r}) = \int \phi_\gamma(\mathbf{r}, E_\gamma) E_\gamma \mu_a(E_\gamma) dE_\gamma, \tag{8.20}$$

where $\phi_\gamma(\mathbf{r}, E_\gamma)$ is the γ-ray flux as a function of position and energy, and $\mu_a(E_\gamma)$ is the linear absorption coefficient. The computation $\phi_\gamma(\mathbf{r}, E_\gamma)$ is complicated by the fact that γ-rays undergo multiple Compton scattering—a problem discussed in Chapter 10. However, in source-free regions outside the core, such as the thermal shield or reactor vessel, the γ-ray flux may be presumed to fall off approximately exponentially, provided the region in question is not too many mean-free paths thick. In this case,

$$q'''(r) = \int -e^{-\mu_a(E_\gamma)r} \phi_{\gamma 0}(E_\gamma) E_\gamma \mu_a(E_\gamma) dE_\gamma. \tag{8.21}$$

Here, $\phi_{\gamma 0}(E_\gamma)$ is the γ-ray flux as a function of energy incident on the region. Equation (8.21) is usually evaluated by dividing the γ-ray spectrum into a number of energy groups. This procedure is illustrated in Section 10.11.

Fission Product Decay Heating

After a few days of reactor operation, the β- and γ-radiation emitted from decaying fission products amounts to about 7% of the total thermal power output of the reactor. When the reactor is shut down, the accumulated fission products continue to decay and release energy within the reactor. This fission product decay energy can be quite sizable in absolute terms, and a means for cooling the reactor core after shutdown must be provided in all reactors except those operating at very low

power levels. If this is not done, the temperature of the fuel may rise to a point where the integrity of the fuel is compromised and fission products are released.

Consider a reactor that has been operating at a constant thermal power P_0 long enough for the concentrations of the radioactive fission products to come to equilibrium. Since the rate of production of the fission products is proportional to the reactor power, it follows that the activity of the fission products at any time after the reactor has been shut down is also proportional to P_0. The ratio $P(t_s)/P_0$, where $P(t_s)$ is the power (rate of energy release) emanating from the fission products at the time t_s after shutdown, is therefore independent of P_0. Figure 8.3 shows this ratio as a function of t_s, in seconds, for a ^{235}U-fueled thermal reactor. The figure is based on a recoverable energy per fission of 200 MeV.

Figure 8.3 can also be used to give values of the fission product decay power for a reactor that is operated for the finite time t_0 and then shut down. The ratio $P(t_0, t_s)/P_0$, where t_s is again the cooling time (reactor off), is then obtained from the expression

$$\frac{P(t_0, t_s)}{P_0} = \frac{P(t_s)}{P_0} - \frac{P(t_0 + t_s)}{P_0}. \tag{8.22}$$

The two terms on the right of this equation are to be found from Fig. 8.3.

If a ^{235}U-fueled reactor contains substantial quantities of ^{238}U, as many of these reactors do, the decay of ^{239}U and ^{239}Np, formed by the absorption of neu-
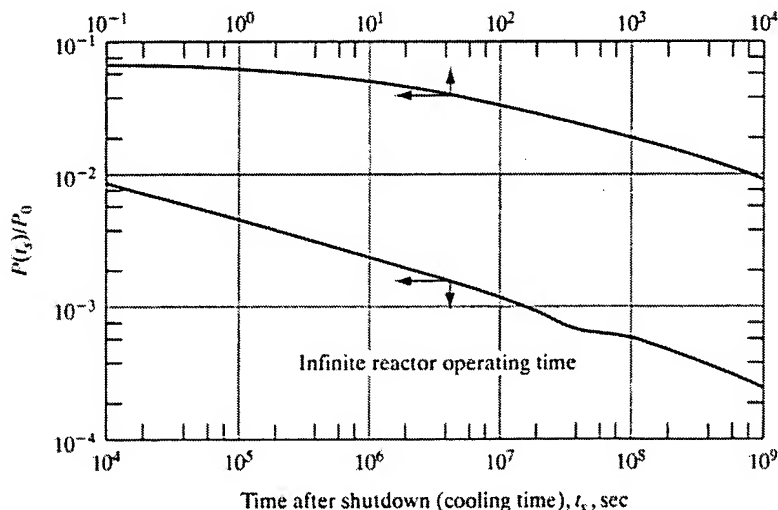

Figure 8.3   The ratio $P(t_s)/P_0$ of the fission product decay power to the reactor operating power as a function of time t_s after shutdown. (Subcommittee ANS-5 of the American Nuclear Society, 1968.)

trons in the ^{238}U, must also be taken into account. Using the equations of radioactive decay derived in Section 2.9, it is easy to show that

$$\frac{P_{29}}{P_0} = 2.28 \times 10^{-3} C \left(\frac{\overline{\sigma}_{a25}}{\overline{\sigma}_{f25}}\right) [1 - e^{-4.91 \times 10^{-4} t_0}] e^{-4.91 \times 10^{-4} t_s} \tag{8.23}$$

and

$$\frac{P_{39}}{P_0} = 2.17 \times 10^{-3} C \left(\frac{\overline{\sigma}_{a25}}{\overline{\sigma}_{f25}}\right) [(1 - e^{-3.41 \times 10^{-6} t_0}) e^{-3.41 \times 10^{-6} t_s}$$
$$- 7.0 \times 10^{-3}(1 - e^{-4.91 \times 10^{-4} t_0}) e^{-4.91 \times 10^{-4} t_s}]. \tag{8.24}$$

In these equations, P_{29} and P_{39} are the decay powers of ^{239}U and ^{239}Np, respectively, C is the conversion factor for the reactor, and $\overline{\sigma}_{a25}$ and $\overline{\sigma}_{f25}$ are the effective thermal cross-sections of ^{235}U. The times t_0 and t_s are again in seconds.

Example 8.4

A certain ^{235}U-fueled reactor operates at a thermal power of 825 MW for 1.5 years and is then shut down. (a) Using Fig. 8.3, compute the decay energy at the following times: at shutdown, 1 hour after shutdown, 1 year after shutdown. (b) If the conversion factor for the reactor is $C = 0.88$, what are the contributions to the decay energy at the prior times due to ^{239}U and ^{239}Np?

Solution.

1. In this problem, $P_0 = 825$ MW and $t_0 = 1.5$ years $= 1.5 \times 3.16 \times 10^7$ sec. Then at shutdown, $t_s = 10^{-1}$ sec (the shortest time in Fig. 8.3), and from Eq. (8.22),

$$\frac{P}{P_0} = \frac{P(10^{-1})}{P_0} - \frac{P(4.74 \times 10^7)}{P_0}.$$

From Fig. 8.3, this is

$$\frac{P}{P_0} = 0.070 - 0.0007 \simeq 0.070,$$

and $P = 825 \times 0.070 = 57.7$ MW. [Ans.] For $t_s = 1$ hr $= 3.6 \times 10^3$ sec:

$$\frac{P}{P_0} = \frac{P(3.6 \times 10^3)}{P_0} - \frac{P(4.74 \times 10^7)}{P_0}$$
$$= 0.014 - 0.0007 \simeq 0.014.$$

so that $P = 825 \times 0.014 = 11.5$ MW. [Ans.] Finally, for $t_s = 1$ yr $= 3.16 \times 10^7$ sec, $t_0 + t_s = 7.90 \times 10^7$ sec:

$$\frac{P}{P_0} = \frac{P(3.16 \times 10^7)}{P_0} - \frac{P(7.90 \times 10^7)}{P_0}$$

$$= 0.00079 - 0.00063 = 0.00016.$$

$$P = 825 \times 0.00016 = 0.132 \text{ MW. } [Ans.]$$

2. If the difference between the non-$1/v$ factors for $\bar{\sigma}_{a25}$ and $\bar{\sigma}_{f25}$ is ignored, $\bar{\sigma}_{a25}/\bar{\sigma}_{f25} = 681/582$. Then with $t_0 = 4.74 \times 10^7$ sec, the exponential in the brackets of Eq. (8.23) is negligible, and with $t_s = 0$ this equation gives

$$\frac{P_{29}}{P_0} = 2.28 \times 10^{-3} \times 0.88 \times \frac{681}{582} = 2.35 \times 10^{-3}.$$

Then $P_{29} = 825 \times 2.35 \times 10^{-3} = 1.95$ MW. [Ans.]
For $t_s = 1$ hr,

$$P_{29} = 1.95 \exp[-4.9 \times 10^{-4} \times 3.600] = 0.33 \text{ MW. } [Ans.]$$

With $t_s = 1$ year, $P_{29} \simeq 0$, as the half-life of ^{239}U is only 23.5 minutes. [Ans.]
For P_{39}, using Eq. (8.24) in a straightforward way, the results are: at shutdown, 1.84 MW; after 1 hour, $\simeq 1.84$ MW; after 1 year, $\simeq$ zero. [Ans.]

8.3 HEAT FLOW BY CONDUCTION

Energy is removed from a reactor by two fundamentally different heat transfer processes—*conduction* and *convection*. In conduction, heat is transmitted from one location in a body to another as a result of a temperature difference existing in the body—there is no macroscopic movement of any portion of the body. It is by this mechanism, as shown in this section, that heat produced in a fuel rod is transferred to the surface of the rod. Heat convection involves the transfer of heat to a moving liquid or gas, again as the result of a temperature difference and the later rejection of this heat at another location. Thus, the heat conducted to the surface of a fuel rod is carried into the coolant and out of the system by convection. Such convective heat transfer is discussed in Section 8.4.

For completeness, it should be mentioned that heat can also be transferred as thermal radiation across a vacuum or other rarefied space between a hotter body and a colder one. However, this process is of relatively little importance except in some gas-cooled reactors and is not considered further.

The Equations of Heat Conduction

The fundamental relationship governing heat conduction is *Fourier's law*, which for an isotropic medium is written as

$$\mathbf{q}'' = -k \text{ grad } T. \tag{8.25}$$

Here $\mathbf{q}''$, which is called the *heat flux*, is defined so that $\mathbf{q}'' \cdot \mathbf{n}$ is equal to the rate of heat flow across a unit area with unit outward normal $\mathbf{n}$. Thus, $\mathbf{q}''$ is entirely analogous to the neutron current density vector defined in Chapter 5. In the English system, $\mathbf{q}''$ has units of Btu/hr-ft^2 and W/m^2 in SI units. The parameter k in Eq. (8.25) is called the *thermal conductivity* and has units of Btu/hr-ft-°F in the English system and W/m-K in SI units. Values of k for a number of important substances are given in Appendix IV. In general, k is a function of temperature. The function T in Eq. (8.25) is the temperature in °F or K as appropriate. The similarity between Eq. (8.25) and Fick's law of diffusion should be especially noted.

Consider an arbitrary volume V of material throughout a portion of which heat is being produced. From the conservation of energy, the net rate at which heat flows out of the surface of V in the steady state, must be equal to the total rate at which heat is produced within V. If this were not the case, the substance would change temperature and therefore would not be in a steady state. In equation form

$$\begin{bmatrix} \text{Net rate of flow} \\ \text{of heat out of } V \end{bmatrix} - \begin{bmatrix} \text{rate of heat production} \\ \text{within } V \end{bmatrix} = 0. \tag{8.26}$$

The net rate at which heat flows out of the surface of V is

$$\text{Heat flow} = \int_A \mathbf{q}'' \cdot \mathbf{n} \, dA, \tag{8.27}$$

where the vector $\mathbf{q}''$ is as defined earlier, $\mathbf{n}$ is a unit vector normal to the surface, and the integral is taken over the entire surface. From the divergence theorem, Eq. (8.27) can also be written as

$$\text{Heat flow} = \int_V \text{div } \mathbf{q}'' dV, \tag{8.28}$$

The total rate of heat production within V is equal to

$$\text{Heat production} = \int_V q''' dV, \tag{8.29}$$

where q''' is the rate at which heat is produced per unit volume.

Equations (8.28) and (8.29) can now be introduced into Eq. (8.26). Since the integrals are over the same arbitrarily selected volumes, their integrands must be equal and the following expression is obtained:

$$\text{div } \mathbf{q}'' - q''' = 0. \tag{8.30}$$

This result is the *steady-state equation of conductivity* for heat transfer and is analogous to the equation of continuity (Eq. 5.15) discussed in Section 5.3 in connection with neutron diffusion. There is no term in Eq. (8.30) equivalent to the absorption

term appearing in the neutron case; heat simply does not vanish within a medium as neutrons are inclined to do.

When Fourier's law, Eq. (8.25), is substituted into Eq. (8.30) and the resulting equation is divided through by k, which is assumed to be constant, the result is

$$\nabla^2 T = \frac{q'''}{k} = 0. \tag{8.31}$$

This is called the *steady-state heat conduction equation* and is of a form known as *Poisson's equation*. In a region where there are no heat sources, $q''' = 0$ and Eq. (8.31) reduces to

$$\nabla^2 T = 0, \tag{8.32}$$

which is called *Laplace's equation*.

These results are now applied to some problems of interest in nuclear reactors. One of the central problems, as is seen, is the calculation of the heat that can be transferred out of a fuel rod and ultimately into a coolant for a given maximum temperature in the fuel. The maximum fuel temperature is a preset condition that must not be exceeded for reasons of safety.

Plate-Type Fuel Elements

Consider first a plate-type fuel element or rod like that shown in Fig. 8.4, consisting of a fueled central strip (the "meat") of thickness $2a$ surrounded on all sides by cladding of thickness b. It is assumed that heat is generated uniformly within the fuel at the rate of q''' Btu/hr-ft^3 or kW/lit, and that the temperature has reached a steady-state distribution throughout the element.

Ordinarily, the total thickness of such an element is small compared with either its width or length. It is reasonable to ignore the negligible amount of heat flowing out through the edges or ends of the element. In short, the heat flows only in the x direction, where x is the distance from the center of the element normal to the surface (see Fig. 8.4). The temperature distribution in the fuel is then determined
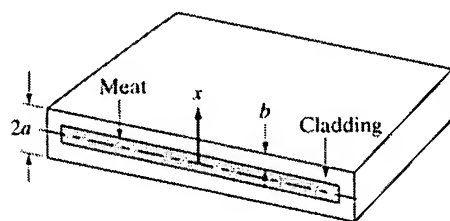

Figure 8.4 A plate-type fuel element.

by Poisson's equation in one dimension,

$$\frac{d^2T}{dx^2} + \frac{q'''}{k_f} = 0, \tag{8.33}$$

where k_f is the thermal conductivity of the fuel.

Two boundary conditions must be specified with a second-order differential equation like Eq. (8.33). In the present problem, these are

$$\text{(i) } T(0) = T_m,$$

where T_m is the maximum (central temperature) in the fuel, and

$$\text{(ii) } \frac{dT}{dx} = 0$$

at $x = 0$. The second condition follows from the symmetry of the problem, which permits no heat flow at the center of the fuel.

Integrating Eq. (8.33) twice gives the general solution

$$T = \frac{q'''}{2k_f}x^2 = C_1x + C_2,$$

where C_1 and C_2 are constants. Placing $x = 0$ immediately gives $C_2 = T_m$. Also, in view of condition (ii), C_1 must be taken to be zero. The temperature within the fuel is therefore

$$T = T_m - \frac{q'''}{2k_f}x^2. \tag{8.34}$$

Using this equation, the temperature T_s at the surface of the fuel (at the fuel-cladding interface) may be evaluated. Thus, writing $x = a$ in Eq. (8.34) gives

$$T_s = T_m - \frac{q'''a^2}{2k_f}. \tag{8.35}$$

The total rate of heat production in the fuel is equal to q''' multiplied by the fuel volume $2Aa$, where A is the area of one face of the fuel. In the steady state, all of the heat produced within the fuel flows out of the fuel. The heat flowing through one face of the fuel is therefore

$$q = q'''Aa. \tag{8.36}$$

This result can also be obtained from Eq. (8.34) using Fourier's law. Thus, the heat flux (heat flow per unit area) is given by

$$q'' = k_f \frac{dT}{dx},$$

where the derivative is to be evaluated at $x = a$. Inserting Eq. (8.34) and performing the differentiation yields

$$q'' = q'''a.$$

The total rate of heat flowing out of one side of the fuel is then

$$q = q''A = q'''Aa,$$

which is the same as Eq. (8.36).

It is sometimes convenient to rewrite Eq. (8.36) in a form in which q''' has been eliminated through Eq. (8.35). Thus, solving Eq. (8.35) for q''' and inserting the result into Eq. (8.36) gives

$$q = \frac{T_m - T_s}{a/2k_f A}. \qquad (8.37)$$

This expression may be viewed as the heat transfer analogue of Ohm's law in electricity—namely.

$$I = \frac{V}{R}. \qquad (8.38)$$

where I is the current, V is the potential difference, and R is the resistance. In the present case, q corresponds to I, $T_m - T_s$ is analogous to the potential difference, and $a/2k_f A$ is called the *thermal resistance*. The value of this analogy is clear shortly.

Turning next to the temperature distribution in the cladding of the plate-type fuel element shown in Fig. 8.4, it is first observed that, since there is little or no heat generated in this region, $q''' = 0$ and the heat conduction equation reduces to

$$\frac{d^2T}{dx^2} = 0. \qquad (8.39)$$

This is the simplest form of Laplace's equation (Eq. 8.32). The boundary conditions are now

(i) $T(a) = T_s$
(ii) $T(a + b) = T_c,$

where T_c is the temperature at the outer surface of the cladding.

Integrating Eq. (8.39) twice gives

$$T = C_1 x + C_2.$$

The constants C_1 and C_2 are easily found from conditions (i) and (ii), and the final expression for T is

$$T = T_s - \frac{x - a}{b}(T_s - T_c). \tag{8.40}$$

It should be observed that T is a linear function of position in the source-free cladding, whereas it is quadratic in the region containing fuel. The temperature distribution in the fuel element is as shown in Fig. 8.5.

Since there are no sources or sinks for heat within the cladding, all of the heat passing into the cladding from the fuel is conducted to the outer surface of the cladding. This rate of heat flow can be found by multiplying the heat flux, which is obtained by applying Fourier's law to Eq. (8.40), by the area of the element. This gives

$$q = \frac{k_c A}{b}(T_s - T_c), \tag{8.41}$$

where k_c is the conductivity of the cladding. If Eq. (8.41) is put in the form

$$T_s - T_c = q\left(\frac{b}{k_c A}\right)$$

and Eq. (8.37) is rewritten as

$$(T_m - T_s) = q\left(\frac{a}{2k_f A}\right),$$

the temperature T_s, which is not a conveniently measured parameter, can be eliminated by adding the last two equations. The result is

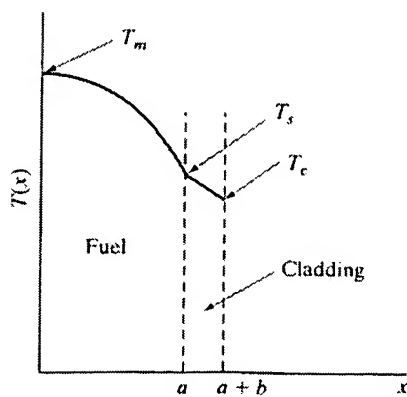$$(T_m - T_c) = q\left(\frac{a}{2k_f A} + \frac{b}{k_c A}\right)$$

Figure 8.5 Temperature distribution across plate-type fuel element.

or

$$q = \frac{T_m - T_c}{\dfrac{a}{2k_f A} + \dfrac{b}{k_c A}}. \tag{8.42}$$

This formula gives the rate at which heat flows through one side of the fuel plate in terms of the difference in temperature between the center and surface of the plate. In the electrical analogue, $T_m - T_c$ is the difference in potential and

$$R = \frac{a}{2k_f A} + \frac{b}{k_c A} \tag{8.43}$$

is the total thermal resistance of the fuel and cladding.

Equation (8.43) shows that, when heat passes through a succession of two materials, the situation is analogous to an electrical potential across two resistances connected in series, where the total resistance is the sum of the two. This conclusion is readily generalized to a sequence of any number of materials—the total thermal resistance is the sum of the resistances of each material.

In the derivation of Eq. (8.42), it is assumed implicitly that the fuel and cladding are tightly bonded together at their point of contact. Some fuel elements, however, have a thin region of bonding material between the fuel and cladding, whereas others have a narrow region of gas between the two. The total thermal resistance is larger in this case and is obtained by adding an appropriate resistance term to Eq. (8.43). The temperature difference $T_m - T_c$ must then be greater for a given heat flow out of the surface of the fuel element.

Cylindrical Fuel Rod

Consider a long cylindrical fuel rod of radius a surrounded by cladding of thickness b. It is assumed again that heat is produced at the constant rate q''' within the rod and that there is no heat released in the cladding.

The temperature in the rod is only a function of the distance r from the axis of the rod, so that in cylindrical coordinates the heat conduction equation is

$$\frac{d^2 T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{q'''}{k_f} = 0. \tag{8.44}$$

The boundary conditions appropriate to this problem are

(i) T is nonsingular within the rod,
(ii) $T(0) = T_m$,

where T_m is the central temperature of the fuel.

It can be readily verified by substitution that the general solution to Eq. (8.44) is

$$T = -\frac{q'''r^2}{4k_f} + C_1 \ln r + C_2,$$

where C_1 and C_2 are constants to be determined. In view of the boundary conditions, it is evident that $C_1 = 0$ and $C_2 = T_m$. Thus, the temperature within the rod is

$$T = T_m - \frac{q'''r^2}{4k_f}. \tag{8.45}$$

The rate at which heat is produced within the rod, and therefore the rate at which it flows out of the rod, is equal to

$$q = \pi a^2 H q''', \tag{8.46}$$

where H is the length of the rod (approximately equal to the height of the core). Solving for q''' and introducing this into Eq. (8.45), evaluated at $r = a$ where the temperature is T_s, then gives

$$q = \frac{T_m - T_s}{1/(4\pi H k_f)}. \tag{8.47}$$

In view of Eq. (8.47), the thermal resistance of the fuel is clearly

$$R_f = \frac{1}{4\pi H k_f}. \tag{8.48}$$

The heat conduction equation for the cladding is

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0,$$

which has the solution

$$T = C_1 \ln r + C_2.$$

Using the boundary conditions

(i) $T(a) = T_s$,
(ii) $T(a + b) = T_c$

to determine the constants C_1 and C_2 gives

$$T = \frac{T_s \ln(a + b) - T_c \ln a - (T_s - T_c)\ln r}{\ln(1 + b/a)}. \tag{8.49}$$

From Fourier's law, the total heat flowing out of the cladding is

$$q = -2\pi(a + b)Hk_c\frac{dT}{dr},$$

evaluated at $r = a + b$. Carrying out the differentiation of Eq. (8.49) yields

$$q = \frac{2\pi Hk_c(T_s - T_c)}{\ln(1 + b/a)}. \tag{8.50}$$

The thermal resistance of the cladding is therefore

$$R_c = \frac{\ln(1 + b/a)}{2\pi Hk_c}. \tag{8.51}$$

Frequently, b is much less than a. In this case, since

$$\ln\left(1 + \frac{b}{a}\right) \simeq \frac{b}{a},$$

Eq. (8.51) can be written as

$$R_c \simeq \frac{b}{2\pi aHk_c}. \tag{8.52}$$

In terms of the overall temperature difference between the center of the fuel and the surface of the cladding, the heat flowing out of the rod is

$$q = \frac{T_m - T_c}{R_f + R_c}, \tag{8.53}$$

where R_f is given by Eq. (8.48) and R_c is given by Eq. (8.51) or (8.52). This result may be verified by solving Eq. (8.47) for $T_m - T_s$ and Eq. (8.50) for $T_s - T_c$ and adding the two expressions.

Space-Dependent Heat Sources

In both of the foregoing calculations of the temperature distribution and heat flow in plate-type and cylindrical fuel rods, it is assumed that the heat is produced at the constant rate q''' throughout the fueled portion of the rod. Accordingly, the temperature does not vary along the length of the rod, and it is possible to relate the total heat flow out of the *entire* rod to the difference in temperature between the center and surface of the rod.

However, it is shown in Section 8.2 that q''' does, in fact, vary approximately as (see Eqs. [8.12] and [8.13])

$$q''' = q'''_{max} \cos\left(\frac{\pi z}{\tilde{H}}\right). \tag{8.54}$$

where z is measured from the midpoint of the rod. There is negligible variation in q''' in the x and y directions of the plate-type element or in the radial direction of a cylindrical rod. A nonuniform heat distribution like that given in Eq. (8.54) would be expected to give rise to a nonuniform temperature distribution along the length of a fuel rod—and, indeed, to some extent this is the case. In a power reactor, however, the coolant passes along the fuel rods at a temperature considerably lower than the temperature of the fuel, and the temperature gradient is therefore much steeper across the diameter of the fuel than along the length of the rod. As a consequence, most of the heat generated in the fuel flows directly to the surface of the rod; there is little or no heat flow in the z direction.

It is possible to generalize the calculations made in this section to the case where q''' is a function of z. This can be done by dividing the calculated value of q, which is the the total heat flow out of the surface of a fuel rod, by the surface area of the rod. This gives the heat flux, q'', which is independent of position if q''' is constant, but varies with position if q''' is not. For instance, if Eq. (8.46) is divided by $2\pi(a+b)H$, the area of a clad fuel rod, the result is

$$q''(z) = \frac{a^2}{2(a+b)} q'''(z). \tag{8.55}$$

This is the heat flux at the surface of the rod as a function of z. Similarly, $q''(z)$ can be found as a function of the temperature difference $T_m - T_c$, where both temperatures may be functions of z, by dividing Eq. (8.53) by $2\pi(a+b)H$:

$$q''(z) = \frac{T_m(z) - T_c(z)}{2\pi(a+b)H(R_f + R_c)}. \tag{8.56}$$

Example 8.5

The fuel rods for the reactor described in Example 8.3 consist of a fueled portion 0.42 in. in diameter that is clad with Zircaloy-4, 0.024 in thick. Each rod is 12 ft long. Given that the center temperature of the fuel is 3,970 °F at the midpoint of the central rod, calculate at this point the (a) heat flux out of the rod; (b) outer temperature of the cladding.

Solution.

1. From Example 8.3, q''' at the midpoint of the central rod is 4.66×10^7 BTU/hr-ft^3. The radius of the fuel is $0.21/12 = 0.0175$ ft, whereas the cladding thickness is $0.024/12 = 0.002$ ft. Then using Eq. (8.55),

(8.54)

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(8.53) by $2\pi(a+b)H$:

)   (8.56)
$+ R_c)$.

le 8.3 consist of a fueled portion
24 in thick. Each rod is 12 ft long.
0 °F at the midpoint of the central
the rod; (b) outer temperature of

central rod is 4.66×10^7 BTU/hr-
75 ft, whereas the cladding thick-
(8.55).

$$q'' = \frac{(0.0175)^2}{2(0.0175 + 0.002)} \times 4.66 \times 10^7$$
$$= 3.66 \times 10^5 \text{ Btu/hr-ft}^2$$
$$= 115.5 \text{W/cm}^2. \quad [Ans.]$$

2. From Table IV.6, $k_f = 1.1$ Btu/hr-ft-°F and $k_c \simeq 10$ Btu/hr-ft-°F. Equation (8.48) then gives for the thermal resistance of the fuel

$$R_f = \frac{1}{4\pi \times 12 \times 1.1} = 6.03 \times 10^{-3} \text{ °F-hr/Btu.}$$

For the cladding, using Eq. (8.51),

$$R_c = \frac{\ln(1 + 0.024/0.21)}{2\pi \times 12 \times 10} = 1.43 \times 10^{-4} \text{ °F-hr/Btu.}$$

The total resistance is then 6.17×10^{-3} °F-hr/Btu.

Introducing the prior parameters into Eq. (8.56) then gives

$$T_c = 3,970 - 2\pi(0.0175 + 0.002) \times 12 \times 6.17$$
$$\times 10^{-3} \times 3.66 \times 10^5 = 650°F. \quad [Ans.]$$

Exponential Heat Sources

It is often necessary to calculate the temperature distribution and heat transmission in reactor shields and pressure vessels in which radiation energy is deposited more or less exponentially. Consider a slab of thickness a whose surfaces are held at the temperatures T_1 and T_2, as shown in Fig. 8.6. If x is measured from the surface as
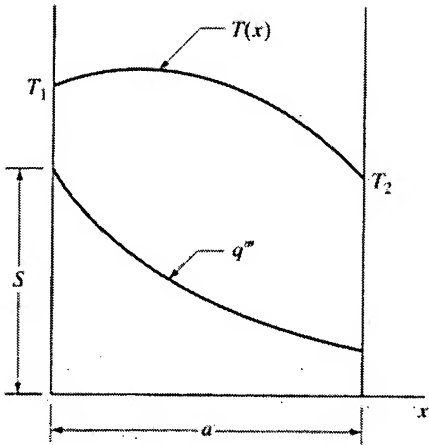

Figure 8.6   Slab containing exponentially distributed heat source.

indicated in the figure, the heat source distribution is given by

$$q''' = Se^{-\mu x},$$ (8.57)

where S and μ are constants. For γ-rays, μ may often be taken to be the linear absorption coefficient.

Introducing Eq. (8.57) into the one-dimensional heat conduction equation (Eq. 8.33) gives

$$\frac{d^2T}{dx^2} + \frac{S}{k}e^{-\mu x} = 0.$$ (8.58)

The general solution to this equation is easily found to be

$$T = C_1 x + C_2 - \frac{S}{k\mu^2}e^{-\mu x},$$

where the constants C_1 and C_2 are found from the boundary conditions

$$T(0) = T_1, \quad T(a) = T_2.$$

The final expression for the temperature distribution is

$$T = T_1 + (T_2 - T_1)\frac{x}{a} + \frac{S}{k\mu^2}\left[1 - e^{-\mu x} - \frac{x}{a}(1 - e^{-\mu a})\right].$$ (8.59)

This equation is the starting point for calculations of heat removal from media in which radiation is absorbed exponentially. It is also used to compute the thermal stresses that accompany such nonuniform temperature distributions. As is shown in the problems at the end of the chapter, under certain conditions the temperature in the slab may rise to a maximum value that is larger than either T_1 or T_2.

8.4 HEAT TRANSFER TO COOLANTS

As noted earlier, the heat produced in the fuel or deposited by radiation in other parts of a reactor is transferred to a coolant of one type or another, which, in turn, carries the heat outside the system. The fundamental relation describing the transfer of heat from a heated solid to a moving fluid (liquid or gas) is *Newton's law of cooling*:

$$q'' = h(T_c - T_b).$$ (8.60)

In this expression, q'' is the heat flux in Btu/hr-ft^2-°F, T_c is the temperature of the surface of the solid (in the case of reactor fuel, this is the outer temperature of the cladding), and T_b is an appropriate reference temperature of the fluid.

The numerical value of the heat transfer coefficient depends on many factors, including the nature of the coolant, the manner in which it flows by the heated surface, and the coolant temperature. Methods for calculating h from experimental data are presented later in this chapter. At this point, however, it is of interest to note the range of values of h that are applicable to the coolants in power reactors. For ordinary and heavy water, h usually lies between 5,000 and 8,000 Btu/hr-ft^2-°F (25 and 45 kW/m^2 K); for gases, h ranges between about 10 and 100 Btu/hr-ft^2-°F (55 and 550 W/m^2 K); and for liquid sodium, h is generally between 4,000 and 50,000 Btu/hr-ft ^{2}-°F (20 and 300 kW/m^2 K).

In most reactor heat transfer problems, the fluid flows along the fuel in well-defined coolant channels. Since the temperature of the fluid ordinarily varies with position across each channel, it is possible to define the temperature T_b in Eq. (8.60) in any number of ways. In most heat transfer calculations, T_b is taken to be the *mixed mean* or *bulk temperature* of the fluid. This is defined by the formula

$$T_b = \frac{\int \rho c_p v T \, dA_c}{\int \rho c_p v \, dA_c}, \tag{8.61}$$

where ρ is the fluid density, c_p is its specific heat, v is the fluid velocity, T is the temperature, dA_c is the differential of channel area, and the integrations are carried out over the cross-section of the channel. In general, each of the variables in the integrals may be a function of position across the channel. The temperature T_b defined by Eq. (8.61) is the temperature the fluid would achieve if it were allowed to mix adiabatically—for instance, if the fluid flowing out of the channel were collected and mixed in an insulated container. Measured values of h are normally given in terms of the bulk temperature of the fluid.

The total rate of heat flow across an area A between a solid and a fluid is

$$q = q'' A = hA(T_c - T_b). \tag{8.62}$$

Written in the form of Ohm's law, Eq. (8.62) is

$$q = \frac{T_c - T_b}{1/ha}. \tag{8.63}$$

The denominator in this equation is the thermal resistance for convective heat transfer,

$$R_h = \frac{1}{hA}. \tag{8.64}$$

It is now possible to return to the fuel element problems considered in the last section and compute the rate at which heat is transferred to a coolant, for a given difference in temperature between the center of the fuel and the fluid. For

the cladded plate-type fuel element, the total thermal resistance is the sum of Eqs. (8.43) and (8.64):

$$R = \frac{a}{2k_f A} + \frac{b}{k_c A} + \frac{1}{hA}, \tag{8.65}$$

and the total heat flow through one side of the fuel element to the fluid is

$$q = \frac{T_m - T_b}{\dfrac{a}{2k_f A} + \dfrac{b}{k_c A} + \dfrac{1}{hA}}. \tag{8.66}$$

Similarly, from Eqs. (8.48) and (8.51), the total thermal resistance for a clad cylindrical fuel rod is

$$R = \frac{1}{4\pi H k_f} + \frac{\ln(1 + b/a)}{2\pi H k_c} + \frac{1}{hA}. \tag{8.67}$$

where $A = 2\pi(a + b)H$. If $b \ll a$,

$$R = \frac{1}{4\pi H k_f} + \frac{b}{2\pi a H k_c} + \frac{1}{hA}. \tag{8.68}$$

In either case, the rate of heat flow into the coolant is

$$q = \frac{T_m - T_b}{R}. \tag{8.69}$$

These results, which apply to the case of uniform heat production along the length of the fuel, can be generalized to the more realistic situation where q''' depends on z. Thus, as explained in the preceding section, most of the heat produced in the fuel flows directly to the coolant in a direction normal to the axis of the fuel rod. The expression for the heat flux, obtained by dividing q in Eq. (8.69) by the surface area of the rod, is therefore valid at every point along the rod. Thus, for the cylindrical rod, it follows that

$$q''(z) = \frac{T_m(z) - T_b(z)}{2\pi(a + b)HR}. \tag{8.70}$$

In terms of T_c, the outer temperature of the cladding,

$$q''(z) = \frac{T_c(z) - T_b(z)}{2\pi(a + b)HR_h}$$
$$= h[T_c(z) - T_b(z)],$$

which is simply Newton's law, Eq. (8.60).

Example 8.6

The fuel rods in Examples 8.3 and 8.5 are cooled with pressurized water. Given that the heat transfer coefficient is 7,500 Btu/hr-ft²-°F, calculate the bulk temperature of the water opposite the midpoint of the hottest rod.

Solution. According to Example 8.5, at the point in question, $q'' = 3.66 \times 10^5$ Btu/hr-ft² and $T_c = 650°F$. From Eq. (8.60),

$$T_b = 650 - 3.66 \times 10^5/7,500$$

$$= 601°F. \text{ [Ans.]}$$

For reasons discussed later, this is not the highest temperature of the coolant, although q''' is greatest at this point.

Temperature along a Coolant Channel

As the coolant moves along the fuel, it absorbs heat; as a result, its temperature continually increases. However, the temperature does not increase at a constant rate since the heat is released from the fuel nonuniformly—according to

$$q''' = q'''_{max} \cos\left(\frac{\pi z}{\widetilde{H}}\right). \tag{8.71}$$

This equation applies to the central ($r = 0$) rod; for other rods, q''' must be multiplied by $J_0(2.405r/\widetilde{R})$. A simple method is now discussed for estimating the variation in the temperature of the coolant and the fuel as a function of z, when the heat is produced in the fuel according to Eq. (8.71).

Before proceeding, it should be noted that, except for the fuel rods at the edge of the core, there is only one coolant channel associated with each fuel rod. This may be taken to be either the volume between neighboring fuel rods, with a portion of each rod contributing heat to the channel, or the volume of coolant in a unit cell surrounding one rod. As shown in Fig. 8.7 for a square array of rods, the volumes of coolant associated with the surface area of one rod are the same in both cases.

Consider a coolant slab of thickness dz as it moves along a coolant channel. The volume of this slab is equal to $A_c dz$, where A_c is the cross-sectional area of the channel, and its mass is $\rho A_c dz$, where ρ is the coolant density. As the slab progresses up the channel the distance dz, it absorbs heat from the fuel, which raises the temperature of the slab the amount dT_b.[6] The heat required to produce the change dT_b is equal to

$$\rho A_c dz c_p dT_b,$$

[6]Throughout this discussion, the coolant is assumed not to boil. If it is boiling, then the absorption of heat goes into vaporization of the coolant at constant temperature.
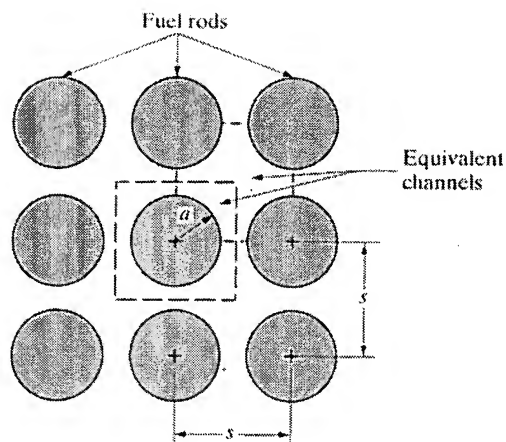
Figure 8.7 Equivalent cooling channels in a square fuel lattice.

where c_p is the specific heat per unit mass. The *rate* at which heat is added to the flowing coolant is obtained by dividing this quantity by the time dt required for the coolant to move the distance dz. Thus,

$$dq = \rho A_c \frac{dz}{dt} c_p dT_b = \rho A_c v c_p dT_b,$$

where v is the velocity of the fluid. However, $\rho A_c v$ is equal to the rate of flow of the coolant through the channel, w, and so

$$dq = w c_p dT_b. \tag{8.72}$$

As noted earlier, most of the heat produced in the fuel flows directly into the coolant and not along the interior of the fuel. Therefore, the heat dq in Eq. (8.72) is generated in a fuel volume of $A_f dz$, where A_f is the cross-sectional area of the fuel portion of a fuel rod. Thus,

$$w c_p dT_b = q''' A_f dz.$$

Introducing the function q''' from Eq. (8.71) and integrating from the point of entry of the coolant at $z = -H/2$, where its temperature is T_{b0}, to an arbitrary point z along the channel gives

$$T_b = T_{b0} + \frac{q'''_{\max} A_f H}{\pi w c_p} \left[1 + \sin\left(\frac{\pi z}{\widetilde{H}}\right) \right]$$

$$= T_{b0} + \frac{q'''_{\max} V_f}{\pi w c_p} \left[1 + \sin\left(\frac{\pi z}{\widetilde{H}}\right) \right]. \tag{8.73}$$

where V_f is the volume of the fueled portion of the rod.

Equation (8.73) gives the temperature of the coolant as a function of position along the central or *hottest* channel. The temperature along other channels can be obtained by multiplying the second term in Eq. (8.73) by the factor $J_0(2.405r/\widetilde{R})$. According to Eq. (8.73), the coolant temperature increases along the channel and reaches a maximum value

$$T_{b,\max} = T_{b0} + \frac{2q_{\max}''' V_f}{\pi w c_p} \tag{8.74}$$

at the exit of the channel. This behavior of T_b is illustrated in Fig. 8.8. The temperature of the fluid leaving channels other than the central, hottest channel is also highest at the exit, but necessarily lower than the temperature of the coolant leaving the hottest channel.

The temperature T_c of the surface of the cladding can now be found as a function of position along the channel by observing that the heat transferred from a length dz of the fuel rod to the coolant is $hC_c dz (T_c - T_b)$, where C_c is the circumference of the clad rod. This is equal to the heat generated in the length dz of the fuel—namely, $q_{\max}''' A_f dz \cos(\pi z/\widetilde{H})$. It follows that

$$hC_c(T_c - T_b) = q_{\max}''' A_f \cos\left(\frac{\pi z}{\widetilde{H}}\right).$$

Inserting T_b from Eq. (8.73) and solving for T_c gives

$$T_c = T_{b0} + \frac{q_{\max}''' V_f}{\pi w c_p}\left[1 + \sin\left(\frac{\pi z}{\widetilde{H}}\right)\right] + \frac{q_{\max}''' A_f}{hC_c}\cos\left(\frac{\pi z}{\widetilde{H}}\right).$$
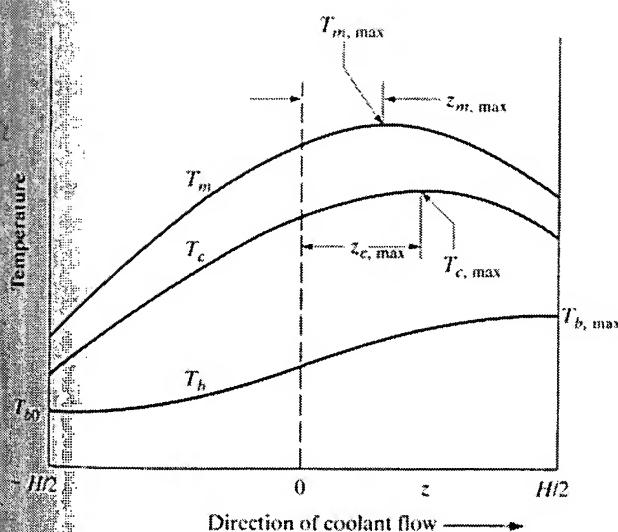

Figure 8.8   Axial temperature of fuel, T_m, temperature of cladding surface, T_c, and bulk temperature of coolant, T_b, as functions of distance along a coolant channel.

This may also be written as

$$T_c = T_{b0} + \frac{q'''_{max} V_f}{\pi w c_p} \left[1 + \sin\left(\frac{\pi z}{\widetilde{H}}\right) \right] + q'''_{max} V_f R_h \cos\left(\frac{\pi z}{\widetilde{H}}\right). \tag{8.75}$$

where R_h is the resistance for convective heat transfer (see Eq. [8.64]).

The temperature T_m along the center of the fuel can also be calculated from Eq. (8.73). It is assumed that all of the heat produced in a slab of fuel of thickness dz flows directly into the coolant. This rate of heat flow is equal to the temperature difference $T_m - T_b$ divided by the total thermal resistance of a section of fuel and coolant dz thick. The resistance of such a section is simply the resistance calculated earlier for the full length of the fuel multiplied by the ratio dz/H. Equating heat production and heat flux then gives

$$\frac{T_m - T_b}{H} = q'''_{max} A_f \cos\left(\frac{\pi z}{\widetilde{H}}\right),$$

where R is given by Eq. (8.65) for plate-type fuel elements and by Eq. (8.67) or Eq. (8.68) for cylindrical fuel rods. Solving for T_m and introducing T_b from Eq. (8.73) yields finally

$$T_m = T_{b0} + \frac{q'''_{max} V_f}{\pi w c_p} \left[1 + \sin\left(\frac{\pi z}{\widetilde{H}}\right) \right] + q'''_{max} V_f R \cos\left(\frac{\pi z}{\widetilde{H}}\right). \tag{8.76}$$

A plot of Eqs. (8.75) and (8.76) is given in Fig. 8.8. It is evident from the figure that both T_c and T_m rise along the channel and reach maximum values in the upper part of the channel beyond the midpoint of the fuel. There are two reasons that the maximum temperature of the fuel occurs there, rather than at midchannel, where q''' is the greatest. First, the temperature of the coolant continues to increase past the midpoint. Second, the heat flux q'' is determined only by the value of q''', and this, being a cosine function, decreases very slowly in the vicinity of $z = 0$. But q'', in turn, specifies the temperature difference $T_m - T_b$. Therefore, with T_b increasing, T_m must also increase to provide the actual value of q''. Further along the channel, q'' begins to drop more rapidly, and T_m eventually decreases. It is this combined effect of a rising T_b and a decreasing q'' that gives rise to and determines the position of the maximum fuel (and cladding) temperature.

To find the locations of these maximum temperatures, the derivatives of T_c or T_m are placed equal to zero. When this is done with Eq. (8.75), the result is easily found to be

$$z_{c,max} = \frac{\widetilde{H}}{\pi} \cot^{-1}(\pi w c_p R_h). \tag{8.77}$$

Similarly, from Eq. (8.76),

$$z_{m,\max} = \frac{\widetilde{H}}{\pi} \cot^{-1}(\pi w c_p R). \tag{8.78}$$

The actual maximum values of T_c and T_m can be found by introducing Eqs. (8.77) and (8.78) into Eqs. (8.75) and (8.76). With a little trigonometry, it is easy to obtain the following results:

$$T_{c,\max} = T_{b0} + q_{\max}''' V_f R_h \left[\frac{1 + \sqrt{1 + \alpha^2}}{\alpha} \right]. \tag{8.79}$$

where

$$\alpha = \pi w c_p R_h \tag{8.80}$$

and

$$T_{m,\max} = T_{b0} + q_{\max}''' V_f R \left[\frac{1 + \sqrt{1 + \beta^2}}{\beta} \right], \tag{8.81}$$

where

$$\beta = \pi w c_p R. \tag{8.82}$$

It must be emphasized that the previous calculations of T_b, T_c, and T_m as functions of z are highly approximate and give only the most qualitative behavior of these temperatures. Furthermore, the results are not applicable to liquid metal coolants, where there may be appreciable heat conduction in the z direction. The derivations also do not apply to coolants that are undergoing bulk boiling, although they are valid for locally boiling coolants.

Example 8.7

Pressurized water enters the core of the reactor described in Examples 8.3 and 8.5 at a temperature of $543°F$ and passes along the fuel rods at the rate of 3,148 lb/hr per channel. (a) What is the exit temperature of the coolant from the hottest channel? (b) What is the maximum temperature of the cladding and the fuel in this channel?

Solution.

1. The volume of the fueled portion of the fuel rod is

$$V_f = 1.15 \times 10^{-2} \text{ft}^2.$$

From Example 8.3, $q_{\max}''' = 4.66 \times 10^7$ Btu/hr-ft^3, and from Table IV.3, c_p has a value of approximately 1.3 Btu/lb-°F. Then using Eq. (8.74),

$$T_{b,max} = 543 + \frac{2 \times 4.66 \times 10^7 \times 1.15 \times 10^{-2}}{\pi \times 3148 \times 1.13}$$

$$= 543 + 83 = 626°F. \; [Ans.]$$

2. From Eq. (8.64) and Example 8.6, the total convective resistance is

$$R_h = 1/7,500 \times 2\pi(0.0175 + 0.002) \times 12 = 9.07 \times 10^{-5} °F\text{-hr/Btu.}$$

The location of the maximum cladding temperature is then given by Eq. (8.77)

$$z_{c,max} = \frac{12}{\pi} \cot^{-1}(\pi \times 3148 \times 1.3 \times 9.07 \times 10^{-5})$$

$$= \frac{12}{\pi} \cot^{-1}(1.17) = 2.71 \; ft.$$

Thus, the maximum value of T_c occurs 2.71 ft beyond the midpoint of the channel. The value of α in Eq. (8.80) is 1.17 and from Eq. (8.79), the maximum cladding temperature is

$$z_{c,max} = 543 + 4.66 \times 10^7 \times 1.15 \times 10^{-2}$$

$$\times 9.07 \times 10^{-5} \times \frac{1 + \sqrt{1 + (1.17)^2}}{1.17}$$

$$= 649°F. \; [Ans.]$$

For the fuel, from Example 8.5, $R_f + R_c = 6.17 \times 10^{-3}$, so the total resistance R is $6.17 \times 10^{-3} + 9.07 \times 10^{-5} = 6.26 \times 10^{-3} °F\text{-hr/Btu.}$ Then using Eqs. (8.78), (8.81), and (8.82), $\beta = 80.5$, $z_{m,max} = 0.047$ ft (so that the temperature of the fuel is greatest at just about the midpoint of the fuel), and $T_{m,max} = 3,940°F.$ [Ans.]

The Heat Transfer Coefficient—Nonmetallic Coolants

The extent to which heat is transferred to a moving fluid, and hence the value of the heat transfer coefficient, depends on the details of the internal motions of the fluid as it flows along a coolant channel. If every portion of the fluid moves parallel to the walls of the channel, then the heat travels radially into the fluid largely by conduction. The flow, in this case, is said to be *laminar*. In contrast, if there are significant radial components of velocity fluctuations within the fluid, the heat is picked up at the wall by portions of the fluid and carried directly into the interior of the channel. This is the description of *turbulent* flow. Clearly, other things being equal, heat is more readily transferred to a fluid undergoing turbulent flow than laminar flow. As a general rule, in those reactors in which the coolant is pumped through the system (as opposed to reactors cooled by natural convection), the coolant flows under turbulent conditions.
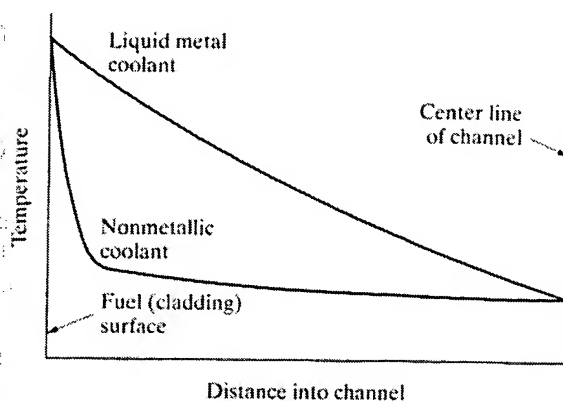
Figure 8.9 Temperature distributions in a nonmetallic coolant and in a liquid metal coolant undergoing turbulent flow.

One of the consequences of the internal motions of the coolant undergoing turbulent flow is that the temperature distribution tends to be more or less uniform over much of the interior region of a coolant channel. Thus, as shown in Fig. 8.9, the temperature drops rapidly with distance in the vicinity of the fuel and quickly reaches the bulk temperature of the fluid. This is in marked contrast to the situation in conduction, where the temperature changes more uniformly with position. Even under turbulent conditions, however, some heat is always transferred to the interior of a coolant by conduction, but for the nonmetallic coolants this contribution is negligibly small. With ordinary water, for example, less than 1% of the heat transfer occurs by conduction when the water is undergoing turbulent flow.

It is possible to characterize the flow of a fluid in terms of a dimensionless parameter known as the *Reynolds number*, which is defined as

$$\text{Re} = \frac{D_e v \rho}{\mu}, \tag{8.83}$$

where D_e is the *equivalent diameter* of the coolant channel discussed later, v is the average velocity of the fluid, ρ is its density, and μ is the fluid viscosity. The value of D_e is to be computed from the formula

$$D_e = 4 \times \frac{\text{cross-sectional area of coolant channel}}{\text{wetted perimeter of coolant channel}}. \tag{8.84}$$

The term *wetted perimeter* in this expression refers to that portion of the perimeter of the channel that is structural and that therefore creates drag to passing coolant. For a hollow pipe carrying a coolant, the wetted perimeter is simply the interior perimeter of a section of the pipe perpendicular to its axis. Thus, with a circular pipe of inside radius a, the numerator in Eq. (8.84) is πa^2 and the denominator is $2\pi a$, so that $D_e = 2a$, the actual pipe diameter. For a bundle of rods of radius a in a square array of pitch s, as shown in Fig. 8.7, the cross-sectional area of a single

coolant channel is equal to $s^2 - \pi a^2$, the wetted perimeter is $2\pi a$, and Eq. (8.84) gives

$$D_e = 2 \times \frac{s^2 - \pi a^2}{\pi a}. \tag{8.85}$$

It has been found experimentally that the flow of most fluids is laminar up to a value of the Reynolds number of about 2,000. Between 2,000 and 10,000, the flow is partly laminar and partly turbulent, the fraction of each depending on the structural details of the coolant channel, the roughness of the channel walls, and other factors. Above $\mathrm{Re} = 10,000$, a fluid moves in fully developed turbulent flow. In summary, a high value of the Reynolds number implies a large amount of turbulence, a high value of the heat transfer coefficient, and a high rate of heat flow into the coolant for a given difference in temperature between the cladding and the coolant.

Example 8.8

The fuel rods described in Example 8.5 are placed in a square array with a pitch of 0.600 in. The rods are cooled by pressurized water ($P = 2,000$ psi), which is flowing at a speed of 15.6 ft/sec. Calculate the Reynolds number for this coolant flow assuming the water temperature is 600°F.

Solution. The radius of the fuel rods is $0.210 + 0.024 = 0.234$ in. From Eq. (8.85), the equivalent diameter is

$$D_e = 2 \times \frac{(0.6)^2 - \pi (0.234)^2}{\pi \times 0.234} = 0.512 \text{ in} = 0.0427 \text{ ft.}$$

The flow velocity is $15.6 \times 3,600 = 56,200$ ft/hr. From Table IV.3, at 600°F and 2,000 psi, $\rho = 42.9$ lb/ft^3 and $\mu = 0.212$ lb/hr-ft. Then substituting into Eq. (8.83) gives

$$\mathrm{Re} = \frac{0.0427 \times 56,200 \times 42.9}{0.212} = 486,000. \text{ [Ans.]}$$

The water is clearly flowing under turbulent conditions.

The numerical value of the heat transfer coefficient h, Eq. (8.60), is a function of the physical properties of the fluid, its rate of flow, and the diameter or effective diameter of the coolant channel. These parameters, including h, can conveniently be grouped together into three dimensionless quantities—the Reynolds number, already discussed; the *Nusselt number*, Nu, defined as

$$\mathrm{Nu} = \frac{h D_e}{k}; \tag{8.86}$$

and the *Prandtl number*, Pr, defined as

$$Pr = \frac{c_p \mu}{k}. \tag{8.87}$$

In these formulas, D_e is the effective diameter of the channel, k and μ are the conductivity and viscosity of the fluid, and c_p is the specific heat. All of these parameters must be specified, of course, in consistent units.

It can be shown both from an analysis of experiments and from theory that convective heat transfer data can be correlated in terms of three dimensionless numbers—Re, Nu, and Pr. In particular, for ordinary water, heavy water, organic liquids, and most gases, all flowing through long straight channels under turbulent conditions, these data can be represented by an equation of the form

$$Nu = C Re^m Pr^n, \tag{8.88}$$

where C, m, and n are constants. The value of h can be obtained from Eq. (8.88) using the definition of the Nusselt number. Thus,

$$h = C \left(\frac{k}{D_e} \right) Re^m Pr^n. \tag{8.89}$$

In using correlations of data of the type expressed by Eqs. (8.88) and (8.89)—or any correlations, for that matter—care must be exercised to determine the reference temperature at which the fluid properties are to be evaluated. Frequently, this is the bulk temperature of the fluid T_b. It should also be noted that these equations are not valid for liquid metals, which must be considered separately.

With ordinary water, heavy water, organic liquids, and gases flowing through long, straight, and *circular* tubes, the following values have been recommended for the constants appearing in Eqs. (8.88) and (8.89): $C = 0.023$, $m = 0.8$, and $n = 0.4$. [With the constants appearing in Eq. (8.88), Eq. (8.88) is known as the *Dittus-Boelter equation*.] These constants are often used in computing h for non-circular coolant channels by introducing the appropriate value of D_e. However, significant errors in h may result if the channels deviate substantially from the circular configuration.

In the important case of ordinary water flowing through a lattice of rods, parallel to the axis of the rods, recommended constants are: $m = 0.8$, $n = 1/3$, and C is given by[7]

$$C = 0.042 \frac{P}{D} - 0.024 \tag{8.90}$$

[7]Weisman, J., "Heat Transfer to Water Flowing Parallel to Tube Bundles," *Nucl. Sci. & Eng.* 6:78, 1979.

for square lattices with $1.1 \leq \frac{P}{D} \leq 1.3$ and

$$C = 0.026\frac{P}{D} - 0.024 \qquad (8.91)$$

for triangular lattices with $1.1 \leq \frac{P}{D} \leq 1.5$. The quantities P and D are, respectively, the lattice pitch and rod diameter.

Example 8.9

Calculate the heat transfer coefficient for the water flowing through the lattice described in Examples 8.5 and 8.8.

Solution. From Example 8.8, the lattice pitch is 0.6 in, the fuel rod radius is 0.234 in, and C is

$$C = 0.042\frac{P}{D} - 0.024 = 0.0299.$$

According to Table IV.3, $c_p = 1.45$ Btu/lb-°F, $\mu = 0.212$ lb/hr-ft, and $k = 0.296$ Btu/hr-ft°F. Introducing these values into Eq. (8.87) gives

$$Pr = \frac{1.45 \times 0.212}{0.296} = 1.039.$$

From Example 8.8, $D_e = 0.0427$ ft and Re = 486,000. Equation (8.89) then gives

$$h = 0.0299 \left(\frac{0.296}{0.0427}\right)(486,000)^{0.8}(1.039)^{1/3}$$

$$= 7436 \text{ Btu/hr-ft}^2\text{-°F. } [Ans.]$$

The Heat Transfer Coefficient—Liquid Metals

Heat transfer to liquid metal coolants is strikingly different from the transfer of heat to ordinary fluids largely because the thermal conductivities of liquid metals are so much higher than those of other types of coolants. At 400°F, for instance, the thermal conductivity of liquid sodium is 46.4 Btu/hr-ft-°F, whereas it is only 0.381 Btu/hr-ft-°F for ordinary water and 0.115 Btu/hr-ft-°F for helium at 1 atm. Thus, at this temperature, the conductivity of sodium is 122 times that of water and 400 times that of helium.

One important effect of the high conductivity of liquid metals is that, even when they are flowing under turbulent conditions, these coolants absorb heat mostly by conduction. This is in marked contrast to the situation with nonmetallic coolants, where, it is recalled, heat transfer occurs largely as the result of the internal motions of the fluid.

Since heat flows into a liquid metal primarily by conduction, the temperature distribution within a coolant channel containing a liquid metal resembles the temperature distribution in a solid conductor whose axis and circumference are held at different temperatures. Thus, as shown in Fig. 8.9, the temperature varies more slowly across the channel with a liquid metal than with a nonmetallic coolant. In both cases, the bulk temperature of the coolant is given by Eq. (8.61).

The heat transfer coefficient for liquid metals has been studied extensively in recent years, and correlations have been given that cover most of the situations encountered in practice. For the case of a liquid metal flowing under turbulent conditions through a hexagonal lattice of rods, parallel to the rods, Dwyer (see References at the end of the chapter) has given the following correlation:

$$\mathrm{Nu} = 6.66 + 3.126(s/d) + 1.184(s/d)^2 + 0.0155(\overline{\Psi}\mathrm{Pe})^{0.86}. \tag{8.92}$$

where s/d is the ratio of lattice pitch to rod diameter, $\overline{\Psi}$ is a function given graphically by Dwyer that has been fitted by Hubbard[8] by the expression

$$\overline{\Psi} = 1 - \frac{0.942(s/d)^{1.4}}{\mathrm{Pr}(\mathrm{Re}/10^3)^{1.281}}. \tag{8.93}$$

and Pe is the so-called *Peclet number*, which is given by

$$\mathrm{Pe} = \mathrm{Re} \times \mathrm{Pr} = \frac{D_e v \rho c_p}{k}. \tag{8.94}$$

Equation (8.92) is valid only for lattices with $s/d > 1.35$. Values of the Nusselt number for more closely spaced hexagonal lattices are given in tabular form by Dwyer, Berry, and Hlavac (see References).

Equations (8.92) and (8.93) may be used for square lattices by replacing the ratio s/d in these expressions by $1.075(s/d)_s$, where $(s/d)_s$ is the value of s/d for the square array. For tightly packed square lattices, the following correlation may be used:

$$\mathrm{Nu} = 0.48 + 0.0133(\mathrm{Pe})^{0.70}. \tag{8.95}$$

8.5 BOILING HEAT TRANSFER

Up to this point, it has been assumed that a liquid coolant does not undergo a change in phase as it moves along a coolant channel absorbing heat from the fuel. It was assumed that the coolant does not boil. However, there are some distinct advantages in permitting a reactor coolant to boil. For one thing, the coolant pressure is much lower when the coolant is allowed to boil than when boiling must be

[8] F. R. Hubbard III, private communication.

prevented. In addition, for a given flow rate and heat flux, lower cladding and fuel temperature are required for a boiling than for a nonboiling coolant. For these reasons, boiling of a restricted nature is now permitted in pressurized water reactors, although steam is not produced directly in these reactors. With boiling water reactors not only is advantage taken of higher heat transfer rates, but, as pointed out in Chapter 4, by producing steam within these reactors, the entire secondary coolant loop of the PWR can be eliminated.

Boiling coolants other than ordinary water have been considered in a number of reactor concepts, but none of these has reached a practical stage of development. Therefore, the following discussion pertains largely to water-cooled reactors. However, the principles to be considered also apply to other types of liquid-cooled reactors.

Boiling Regimes

To understand the phenomenon of boiling heat transfer, consider an experiment in which the heat flux (Btu/hr-ft^2 or W/cm^2) from heated fuel rods to a flowing liquid coolant is measured as a function of the temperature of the surface of the rods for a given system pressure and flow rate. The results of such an experiment are shown in Fig. 8.10. As indicated in the figure, the heat flux increases slowly as the rod
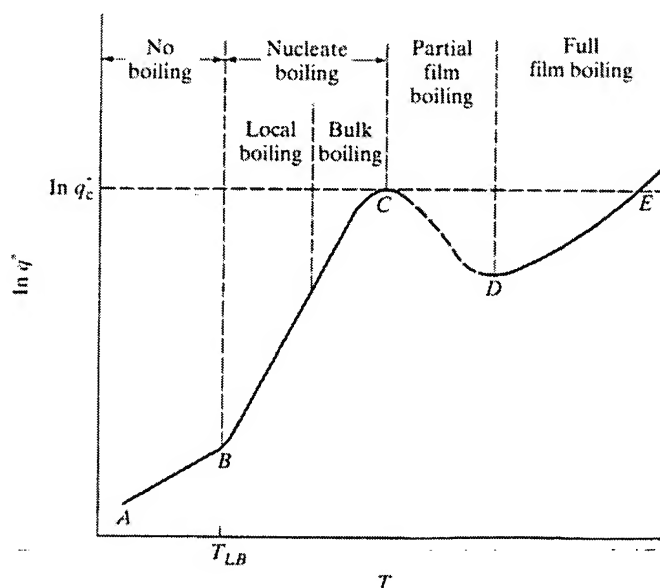

Figure 8.10   The logarithm of the heat flux into a flowing coolant as a function of the surface temperature of the coolant channel.

temperature is increased at low values. In this temperature range, between points A and B in Fig. 8.10, heat is transferred to the coolant by ordinary convection with no change in phase, and the heat transfer coefficient is determined by the correlations given in the preceding section.

As the surface temperature of the fuel is increased further, a point is eventually reached where bubbles of vapor begin to form at various imperfections on the surface of the fuel rods. This occurs at about the point B in Fig. 8.10, and is a form of boiling called *nucleate boiling*. As the bubbles are formed, they are carried away from the rods and into the body of the coolant as a result of the turbulent motions of the fluid. However, as long as the bulk temperature of the coolant is less than its saturation temperature, the vapor in the bubbles soon condenses to the liquid state and the bubbles disappear from the coolant. There is no net production of steam under these circumstances, and the boiling process is termed *subcooled nucleate boiling* or *local boiling*. If and when the bulk temperature of the coolant reaches its saturation temperature, the bubbles persist within the coolant stream, there begins to be a net output of steam, and the system is now said to be undergoing *saturated nucleate boiling* or *bulk boiling*.

The onset of local boiling and the transition to bulk boiling are shown in the lower two portions of Fig. 8.11 for a fluid flowing vertically through a heated pipe.
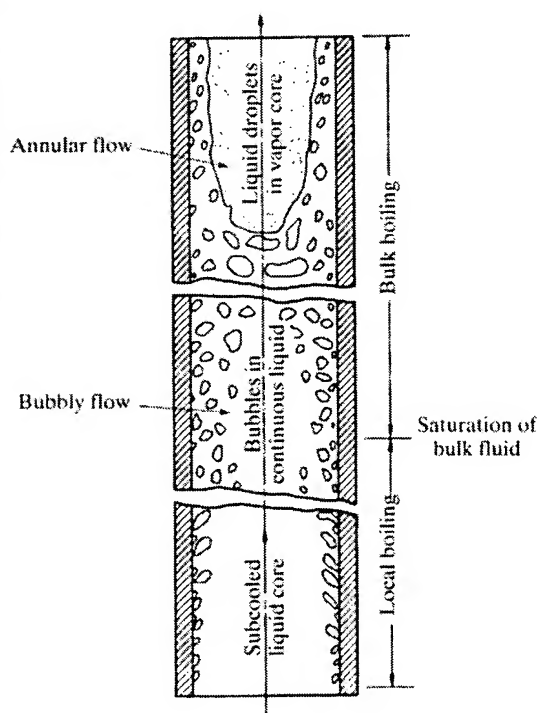

Figure 8.11 Flow patterns in a vertical heated channel. (From L. S. Tong, *Boiling Heat Transfer and Two-Phase Flow.* New York: Wiley, 1967.)

With local boiling, as indicated in the figure, bubbles exist only near the surface of the pipe, and most of the pipe is filled with liquid. In the region with bulk boiling, however, the bubbles are distributed throughout the fluid, which is said to move in *bubbly flow*. Under certain conditions—namely, at high-flow velocities and a large concentration of bubbles (large void fractions)—the bubbles combine to form a void space along the center of the channel. This is called *annular flow*.

In any event, with the onset of nucleate boiling, heat moves readily into the liquid. At every temperature in this region between B and C in Fig. 8.10, heat transfer is more efficient than by ordinary convection. There are two reasons for this. First, heat is removed from the rods both as heat of vaporization and as sensible heat. Second, the motions of the bubbles lead to rapid mixing of the fluid. The rapid increase in the heat flux with temperature, shown by the steep slope of the nucleate boiling region in Fig. 8.10, is explained by the fact that the density of bubbles forming at and departing from the rod surface increases rapidly with surface temperature.

For ordinary water at pressures between 500 and 2,000 psia undergoing nucleate boiling, either local or bulk, the heat flux can be computed from the following correlation:[9]

$$T_c - T_{\text{sat}} = \frac{60(q''/10^6)^{1/4}}{e^{P/900}}. \tag{8.96}$$

where T_c is the surface temperature of the cladding (°F), T_{sat} is the saturation temperature (°F), P is the system pressure (psia), and q'' is the heat flux in Btu/hr-ft². Equation (8.96) is known as the *Jens and Lottes correlation* and is valid for any channel geometry. This correlation is not valid, however, if the coolant is undergoing annular flow.

It is important to estimate the fuel surface temperature T_{LB} at which local boiling begins to know which correlation to use for calculating heat flow into the coolant—a correlation for convective heat transfer or the Jens and Lottes equation. Although there is no single, fixed temperature at which boiling starts, T_{LB} is usually computed from the simple formula

$$T_{\text{LB}} = T_{\text{sat}} + (T_c - T_{\text{sat}})_{\text{JL}} - \frac{q''}{h}. \tag{8.97}$$

In this equation, T_{sat} is again the saturation temperature, $(T_c - T_{\text{sat}})_{\text{JL}}$ is the difference between the cladding and saturation temperatures as computed from the Jens and Lottes correlation, q'' is the heat flux, and h is the convective heat transfer coefficient.

The application of some of the prior results is illustrated in Fig. 8.12, which shows the parameters of the water coolant in a boiling-water reactor as a function

[9] W. H. Jens and P. A. Lottes, USAEC Report ANL-4627, 1951.

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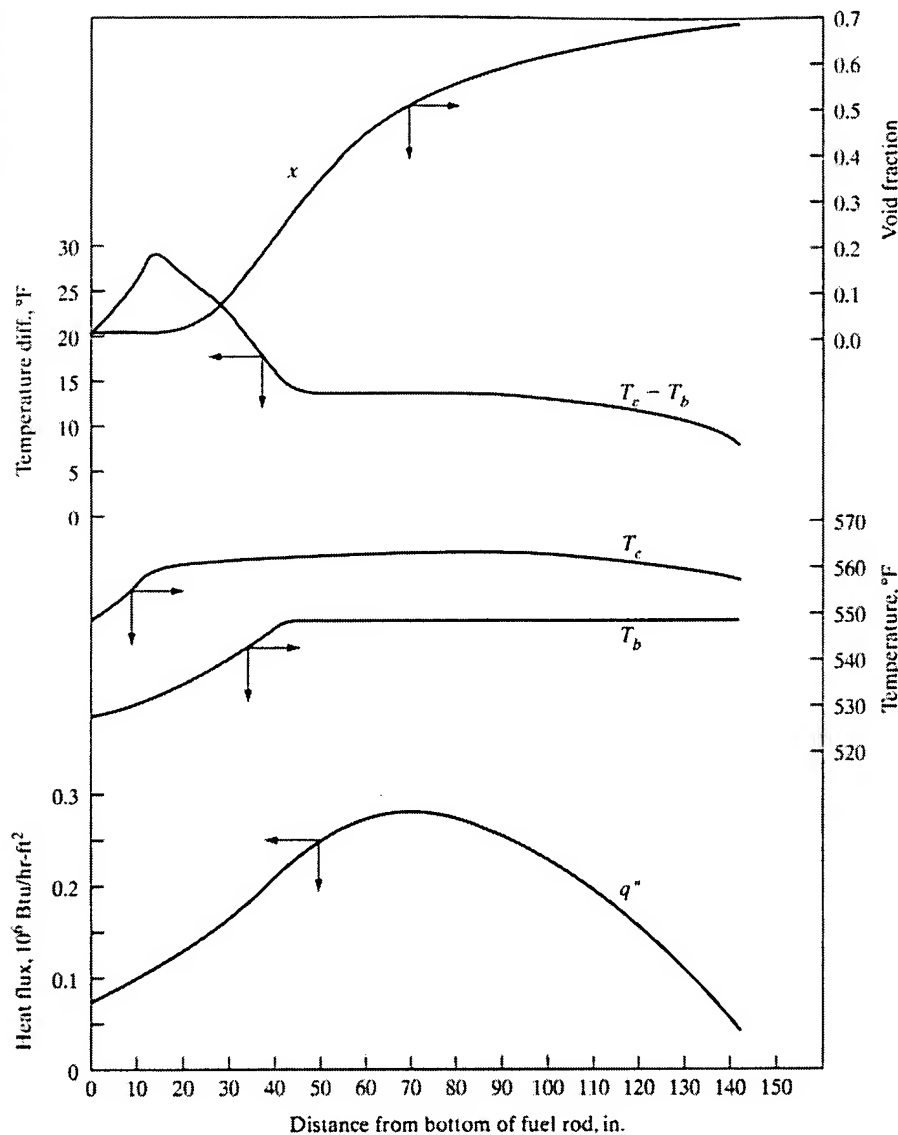ig. 8.12, which
or as a function

Figure 8.12   Thermal analysis of the hottest channel of a BWR. Here, x refers to void fraction not quality. (Courtesy of F. R. Hubbard III.)

of distance up the hottest channel. In this reactor, the fuel rods are about 12 ft long and the system pressure is 1,035 psia. At this pressure, the saturation temperature of water is 548.7°F, which can be estimated from the data in Table IV.2. The lower curve shows the actual heat flux into the channel. The skewing of q'' away from

the bottom of the core is due to the presence of control rods in this region. The calculated value of T_{LB} and the outer cladding temperature T_c are the same—namely, 558°F approximately 13 in up the channel. Local boiling commences at this point. The bulk temperature of the water increases steadily from its entering value of 526°F at the bottom of the core and reaches the saturation temperature of 548.7°F about 45 in along the channel. It is at this point that bulk boiling begins. Thus, the heat transfer is convective for about the first 13 in up the channel, local boiling occurs between 13 in and 45 in, and throughout the rest of the channel the water undergoes bulk boiling. (The flow never becomes annular.)

It should be observed in Fig. 8.12 that the difference in temperature between the cladding and the coolant, after rising in the convective region in response to an increasing q'', drops sharply after the onset of nucleate boiling at 13 in, although q'' is still increasing. This illustrates how readily heat flows into a boiling coolant.

It should also be noted in Fig. 8.12 that the void fraction of the coolant starts to rise from zero at about the point where local boiling begins, although there is no net production of steam until the onset of bulk boiling further up the channel. The reason for this is that, although the steam bubbles that form at the rod surface collapse as they pass into the interior of the channel, their place at the surface is immediately taken by other bubbles. As a consequence, there is always a steady-state distribution of voids across the channel.

The Boiling Crisis

To return to the discussion of the experiment depicted in Fig. 8.10, if the temperature of the fuel surface is increased in the nucleate boiling region, the density of bubbles at or near the surface of the fuel rods also increases. Eventually, however, a point is reached where the bubble density becomes so great that adjacent bubbles coalesce and begin to form a vapor film across the surface of the rods. At this point, which corresponds to the point C in Fig. 8.10, the system is said to be in a *boiling crisis* or in a condition leading to a *departure from nucleate boiling* (DNB). The heat flux at or just before the boiling crisis is called the *critical heat flux*, (CHF) and denoted as q_c''. Sometimes, for reasons that are evident momentarily, q_c is called the *burnout flux*.

With the onset of the boiling crisis, the heat flux into the coolant begins to drop. This is due to the fact, that over the regions of the rods covered by vapor film, the heat is forced to pass through the vapor into the coolant by conduction and radiation, both of which are comparatively inefficient mechanisms for heat transfer. The heat flux continues to drop, more or less erratically (as indicated by the dotted lines in Fig. 8.10), with increasing fuel temperatures as the total area of the film covering the fuel increases. In this region of Fig. 8.10, the system is said to be experiencing *partial film boiling*.

Eventually, when the rod surface temperature is high enough, the vapor film covers the entire rod and the heat flux to the coolant falls to a minimum value (point D). Beyond this point, any increase in temperature leads to an increase in the heat flux simply because heat transfer through the film, although a poor and inefficient process, nevertheless increases with the temperature difference across the film. The system, in this case, is said to be undergoing *full film boiling*.

The existence of the various boiling regimes and the boiling crisis is an important consideration in the design of a liquid-cooled reactor, as may be seen from the following example. Suppose that a water-cooled reactor is designed and/or operated in such a manner that at some point along a coolant channel the water undergoes nucleate boiling near DNB conditions. If the reactor power is suddenly increased so that the heat flux into the water rises above the DNB value of q_c'', partial film boiling will immediately begin in this channel. However, as explained previously, the formation of the film impedes the transfer of heat to the coolant. As a consequence, the heat confined, so to speak, within the fuel raises the fuel temperature and the surface temperature of the rods—forming the channel. This, in turn, leads to an increase in the area of the film, which leads to a further decrease in the heat flux, a further increase in rod surface temperature, and so on. In this way, the wall temperature rapidly increases along the boiling curve from point C to point E. Long before E is reached, however, the temperature of the fuel will attain such high values (several thousands of degrees Fahrenheit) that the fuel will partially melt, the cladding will rupture, and fission products will be released into the coolant. As noted earlier, these are occurrences that must be prevented at all costs. For this reason, it is important to know the value of q_c'' and to keep a reactor from operating near the DNB point.

It must not be implied from Fig. 8.10 that it is necessary to have saturated nucleate boiling (bulk boiling) before the onset of the boiling crisis. The figure was intended merely to show that there are two types of nucleate boiling and that it is possible to make the transition from subcooled to saturated boiling if the bulk temperature of the coolant exceeds the saturation temperature. Bubbles at the surface of the fuel can combine to form areas of vapor film, the beginning of the boiling crisis, although the bubbles would condense if they traveled to the interior of the coolant stream.

A great many correlations have been developed from data on q_c'' (see Tong in the References for a tabulation of these correlations). As would be expected, these correlations differ depending on whether the boiling crisis is reached from subcooled or bulk boiling conditions. For subcooled boiling, the following correlation by Jens and Lottes[10] has been widely used:

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[10] W. H. Jens and P. A. Lottes, *ibid.*

TABLE 8.1 PARAMETERS FOR JENS AND LOTTES CORRELATION

P (psia)	C	m
500	0.817	0.160
1,000	0.626	0.275
2,000	0.445	0.500

$$q_c'' = C \times 10^6 \left(\frac{G}{10^6}\right)^m \Delta T_{sub}^{0.22}, \tag{8.98}$$

where C and m are pressure-dependent parameters given in Table 8.1, G is the coolant mass flux in lb/hr-ft^2, and ΔT_{sub} is the difference between the saturated and local temperatures in °F.

Another correlation for subcooled boiling was obtained by Bernath.[11] This is a combination of the three equations:

$$q_c'' = h_c(T_{wc} - T_b) \tag{8.99}$$

$$T_{wc} = 102.6 \ln P - \frac{97.2 P}{P + 15} - 0.45v + 32 \tag{8.100}$$

$$h_c = 10,890 \left(\frac{D_e}{D_e + D_i}\right) + \frac{48v}{D_e^{0.6}}. \tag{8.101}$$

In these expressions, T_{wc} is the wall (cladding) temperature at the onset of the boiling crisis, T_b is the bulk temperature, P is the pressure in psia, v is the coolant velocity in ft/sec, D_e is the equivalent diameter in feet, and D_i is defined as the heated perimeter of a channel in feet divided by π. The Bernath correlation is valid for pressures between 23 and 3,000 psia, fluid velocities between 4.0 and 54 ft/sec, and for D_e between 0.143 and 0.66 in.

Example 8.10

For the PWR lattice described in Examples 8.3 through 8.8, calculate for the onset of the boiling crisis the: (a) cladding temperature, (b) heat transfer coefficient, and (c) critical heat flux.

Solution.

1. Using Eq. (8.99) of the Bernath correlation,

$$T_{wc} = 102.6 \ln 2,000 - \frac{97.2 \times 2,000}{2,000 + 15} - 0.45 \times 15.6 + 32 = 708°F. \; [Ans.]$$

[11] Bernath, L., *Transactions A.I.Ch.E.*, 1955.

2. From Example 8.8, $D_e = 0.0427$ ft. The heated perimeter of the channel is $2\pi a$, where $a = 0.234$ in. Thus,

$$D_i = 2 \times 0.234/12 = 0.039 \text{ ft.}$$

Then from Eq. (8.101),

$$h_c = 10{,}890 \times \frac{0.0427}{0.0427 + 0.039} + \frac{48 \times 15.6}{(0.0427)^{0.6}}$$

$$= 10{,}659 \text{ Btu/hr-ft}^2\text{-}°\text{F. } [Ans.]$$

3. With a bulk water temperature of about 600°F from Example 8.6, the critical heat flux is

$$q_c'' = 10{,}659 \times (708 - 600) = 1.15 \times 10^6 \text{Btu/hr-ft}^2\text{-}°\text{F. } [Ans.]$$

Equations (8.89) through (8.101) provide estimates of the curve that best represents the mass of necessarily rather scattered heat transfer data. The curve passes more or less through the center of gravity of the data points. In the case of the approach to DNB from bulk boiling conditions, workers at the General Electric Company (GE) have developed correlations that form an envelope of the lowest measured values of q_c''. With bulk boiling, such correlations depend on the *flow quality* of the coolant denoted by the symbol χ, which is defined as

$$\chi = \frac{\text{mass flow rate of vapor (lb/hr-ft}^2)}{\text{mass flow rate of vapor-liquid mixture (lb/hr-ft}^2)}. \tag{8.102}$$

The GE correlation, due to Janssen and Levy,[12] is as follows:

$$\frac{q_c''}{10^6} = 0.705 + 0.237(G/10^6), \quad \chi < \chi_1 \tag{8.103}$$

$$\frac{q_c''}{10^6} = 1.634 - 0.270(G/10^6) - 4.710\chi, \quad \chi_1 < \chi < \chi_2 \tag{8.104}$$

$$\frac{q_c''}{10^6} = 0.605 - 0.164(G/10^6) - 0.653\chi, \quad \chi_2 < \chi, \tag{8.105}$$

where

$$\chi_1 = 0.197 - 0.108(G/10^6), \tag{8.106}$$

$$\chi_2 = 0.254 - 0.026(G/10^6), \tag{8.107}$$

[12]E. Janssen and S. Levy, General Electric Company Report APED-3892, 1962. The General Electric Company has also published a portion of a more recent (1966) correlation due to Hench and Levy.

and G is again the mass flux in lb/hr-ft^2. These equations refer to a system pressure of 1,000 psia. For other pressures, $q_c^{''}$ can be found from

$$q_c^{''}(P) = q_c^{''}(1,000 \text{ psia}) + 400(1,000 - P). \tag{8.108}$$

The GE correlation is valid for pressures between 600 and 1450 psia, G from 0.4 $\times$ 10^6 to 6.0 $\times$ 10^6 lb/hr-ft^2, values of χ up to 0.45, D_e from 0.245 to 1.25 in, and for coolant channels between 29 and 108 in. in length.

All of the foregoing correlations for the critical heat flux were developed largely from data for circular pipes. When used to determine $q_c^{''}$ for noncircular channels or for coolant flowing through lattices of fuel rods, these correlations can be counted on to provide only the roughest approximations for $q_c^{''}$. Various correction factors for the various geometries have been developed, however, but these are too detailed to be discussed here (see Tong in the References).

8.6 THERMAL DESIGN OF A REACTOR

As pointed out earlier in this chapter, reactors must be designed in such a way that the fission products remain confined within the fuel at all times—throughout the operational lifetime of the core, during shutdown, and under accident conditions, when the fuel may be denied normal cooling. This places upper limits on the temperature of the fuel and/or its cladding. The currently accepted design criterion is that the integrity of the cladding must be maintained through all operating conditions. Since the expansion of the fuel on melting can rupture the cladding, this design criterion is essentially equivalent to the requirement that the fuel must not melt.

The melting point of UO$_2$ which depends somewhat on burnup—that is, on the fraction of the fissile atoms that have undergone fission—is generally between 5,000°F and 5,100°F (2,760 °C and 2,815°C) for commercially available UO$_2$. In most reactors fueled with UO$_2$, the maximum fuel temperature is somewhat below 4,500°F (2,480°C).

In the HTGR, where the fuel consists of small particles of uranium and thorium dicarbide in a carbonaceous binder, the maximum permitted fuel temperature is about 6,500°F (3,600°C).

Natural or enriched uranium metal melts at 2,070°F (1,132°C), but it also undergoes two changes in solid phase (alteration in crystalline structure)—first at 1,234°F (668°C) and again at 1,425°F (774°C). However, above about 750°F (400°C), the strength of the metal decreases rapidly. This permits fission product gases to gather and diffuse into pockets in the fuel, which in turn leads to the expansion of the fuel and to cladding failure. Therefore, it is usual practice to design uranium metal-fueled reactors with maximum fuel temperatures below 750°F. This

low temperature is not as serious a drawback in transferring heat to the coolant as it would at first appear since the thermal conductivity of uranium metal is much higher than, say, that of UO_2. At 600°F, for example, k_f is 6.5 times higher for uranium than for UO_2.

The DNB Ratio

To prevent the penetration of the cladding at any point in a water-cooled reactor due to the onset of film boiling, this type of reactor must be designed so that the heat flux q'' is always below the critical (burnout) heat flux q_c''. For this purpose, it is convenient to define the *DNB ratio* as

$$\text{DNBR} = \frac{q_c''}{q_{\text{actual}}''}. \tag{8.109}$$

In this ratio, q_c'' is the critical heat flux computed as a function of distance along the hottest coolant channel from the appropriate correlation given in the preceding section, and q_{actual}'' is the actual surface heat flux at the same position along this channel. If heat flow parallel to the fuel can be ignored, then

$$q_{\text{actual}}'' = \frac{q''' A_f}{C_c}, \tag{8.110}$$

where q''' is the heat production per unit volume, A_f is the cross-sectional area of the fuel, and C_c is the circumference of a heated rod. These two functions and the DNB ratio are shown in Fig. 8.13 for a 1,500-MW boiling-water reactor. BWRs are currently designed with a minimum DNBR of 1.9.[13] By setting this lower limit on the DNB ratio, there can be reasonable certainty that burnout conditions will never be reached anywhere in the reactor, even during transient, overpower situations. The minimum design value for the DNBR for PWRs is 1.3. Needless to say, the establishment of a minimum DNB ratio provides a major limitation on the design of water-cooled reactors.

Hot Channel, Hot Spot Factors

With water-cooled reactors, the maximum heat flux anywhere in the core is limited by the DNB ratio. In gas-cooled reactors, the maximum q'' is determined solely by the requirement that the fuel temperature remain well below the melting point for this value of q_{max}''. In *any* reactor, the extent to which q_{max}'' exceeds the average heat flux in the core is given by the *hot channel factor*, also called the *hot spot factor*.

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[13]The minimum DNB ratio is also called the *minimum critical heat flux ratio* (MCHFR).
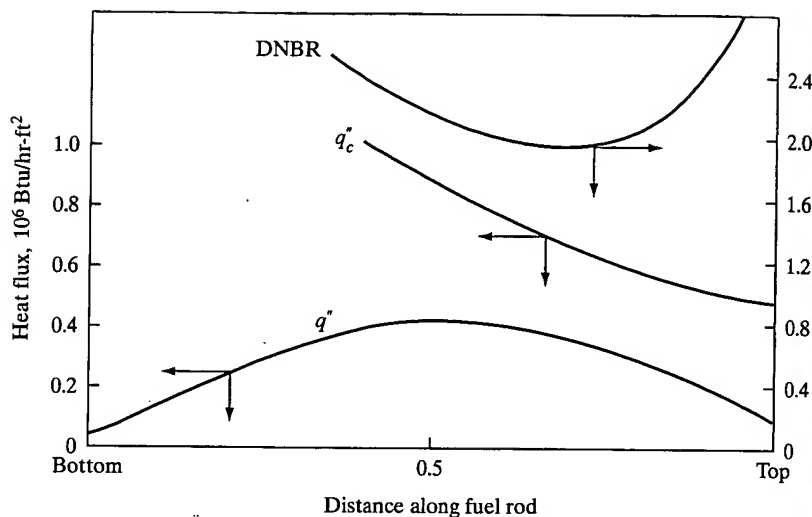
Figure 8.13 The actual heat flux, the computed critical heat flux, and the DNB ratio for a boiling-water reactor. (Courtesy of General Electric Company.)

This is defined by the relation
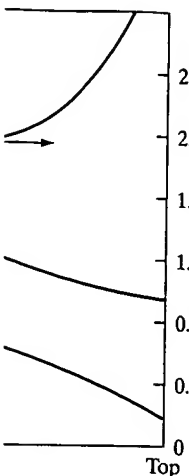
$$F = \frac{q''_{max}}{q''_{av}},\tag{8.111}$$

where q''_{av} is the average heat flux in the core.

There are several reasons that q''_{max} differs from q''_{av}, and therefore why F is different from unity. The most important of these is that, as seen earlier, the power distribution across the core is not flat. If this were the only reason, F could be computed directly for any given reactor design in more or less the same way as the maximum-to-average flux ratio Ω was calculated in Chapter 6. Thus, q''_{max} is a function of various nuclear parameters that determine the power distribution, and q''_{av} is given by the formula

$$q''_{av} = P/A,\tag{8.112}$$

where P is the thermal power output of the core and A is the total heat transfer area of the fuel (with cladding). The previous computation yields what is known as the nuclear hot channel factor F_N.[14]

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[14]For a uniformly fueled reactor at the beginning of core life and in the absence of control rods, F_N is the same as Ω.

In addition to the effect of the nonflat power distribution, q''_{max} may differ from the computed value of q''_{max} as the result of various statistical factors over which the reactor designer has little or no control. For example, the amount of fissile material included in the fuel pellets of a BWR or PWR at the time of manufacture varies slightly from pellet to pellet because of the inherently statistical nature of the manufacturing process. Pellets containing more fissile material produce more power. If such a pellet was located at the point where q'' is highest, the value of q''_{max} would be higher than calculated.

In a similar way, manufacturing tolerances in fuel assemblies may result in slight bowing of the fuel rods, leading to reduced coolant flow and excessive heating of a portion of the fuel. Similarly, fluctuations in the thickness of the cladding may give rise to hot spots where the cladding is thinnest.[15] Also, certain aspects of the coolant flow are inherently statistical in character and tend to give fluctuations in the heat flux.

These various mechanisms, and others by which q''_{max} can differ from its computed value, taken together are described by the *engineering hot channel factor F_E*. The overall hot channel is then

$$F = F_N F_E. \tag{8.113}$$

Each of the individual mechanisms is described, in turn, by an engineering hot channel *factor, $F_{E,x}$*.

The engineering subfactors are obtained from data on fabricated reactor components or from tests, such as coolant flow tests on assembled portions or mockups of the reactor. Consider, for instance, the amount of fissile material m_l per unit length of the fuel rods. For a given thermal flux, q''_{max} is very nearly proportional to m_l. When measurements of m_l are carried out on manufactured rods, it is found that the actual values of m_l form a normal distribution about some mean or average value $\overline{m}_l$ like that shown in Fig. 8.14. With such a distribution, all values of m_l are presumably possible. However, specific values of m_l become less and less likely with increasing deviation from the mean. In particular, it can be shown that the probability that m_l exceeds $\overline{m}_l$ by more than 3σ, where σ is the standard deviation of the distribution, is only 1.35 per 1,000—an unlikely occurrence.

In those situations like the one just described, in which q''_{max} is proportional to a normally distributed engineering variable x, the engineering subfactor is defined as

$$F_{E,x} = \frac{\overline{x} + 3\sigma(x)}{\overline{x}} = 1 + \frac{3\sigma(x)}{\overline{x}}. \tag{8.114}$$

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[15]This is because thin cladding leads to a larger gap (and hence a larger ΔT between the fuel pellet and the cladding).
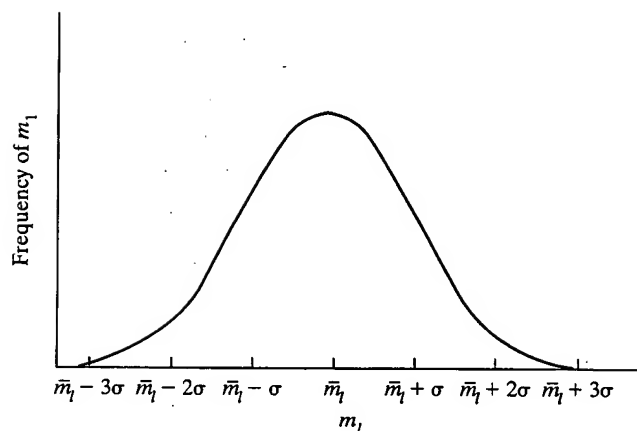
Figure 8.14 Normal distribution of measured values of m_l.

Here $\overline{x}$ is the mean value of x and $\sigma(x)$ is the standard deviation in the measurement of x. For example, in the case of the amount of contained fissile material, the subfactor is

$$F_{E,m_l} = 1 + \frac{3\sigma(m_l)}{\overline{m}_l}. \tag{8.115}$$

Example 8.11

In a certain PWR, the average linear density $\overline{m}_l$ of UO_2 in a fuel rod is 0.457 lb/ft. The standard deviation in the measured values of m_l is 0.0122 lb/ft. What is F_{E,m_l}?

Solution. It is merely necessary to substitute these values into Eq. (8.115). This gives

$$F_{E,m_l} = 1 + \frac{3 \times 0.0122}{0.457}$$

$$= 1.08. \ [Ans.]$$

This result means that, due to statistical fluctuations in the amount of fissile material included in fuel rods during manufacture, there are 1.35 chances in 1,000 that q''_{max} will exceed its computed value by more than 8%.

Calculations of other engineering subfactors, especially those that do not involve a direct proportionality between q''_{max} and the statistical variable, are more complicated and require an analysis of the relationship between the variable in question and q''_{max}. As noted earlier, such subfactors are often obtained from engineering test data.

Once the various engineering subfactors have been determined, it is possible to calculate F_E. This can be done in a number of ways. The most obvious thing is to

multiply all of the subfactors together, and this was done in the case of some early reactors. However, the resulting value of F_E leads to an unnecessarily conservative reactor design since, in using the product of subfactors, it is implicitly assumed that several statistically unlikely events occur simultaneously. The preferred method for computing F_E is based on an analysis of overlapping statistical distributions that is too complex to be reproduced here (see Tong and Weisman in the References). Needless to say, with $F_{E,x}$ known, F_E can be computed.

Reactor Design

In designing a reactor, there is considerable overlap between those aspects of the problem that are specifically nuclear—the neutron flux and power distributions, transient behavior, and so on—and those factors involving the removal of heat from the system. The final reactor necessarily represents a series of compromises between nuclear and thermal-hydraulic factors, all made in the interest of safety and economy. One noted reactor designer frequently refers to reactor design as "the art of compromise."[16]

Of course, most of the reactors manufactured today are not designed from scratch. They are either identical to or improved versions of existing plants. Only rarely is an engineer called on to design a reactor from the beginning. Nevertheless, the several steps in such a design study are now considered to illustrate some of the concepts developed earlier.

Consider the design of a specified type of power reactor having a given thermal power output. The first problem is the selection of the core materials. These are usually chosen on the basis of previous experience, availability, and cost. The amount and enrichment of the fuel are determined in a preliminary way by various nuclear considerations, including estimates of core lifetime and radiation damage to the fuel elements.

With this preliminary choice of materials, a fuel lattice arrangement is next proposed. The operating parameters of the reactor—the neutron flux, the power and temperature distributions, the coolant density and/or void distribution, the temperature and heat flux along the hot channel—are then computed for the proposed lattice. It should be noted that such computations are inherently iterative, at least for some types of reactors. With the PWR or BWR, for instance, it is necessary to know the water density as a function of position to compute the neutron flux and, from this, the power distribution. However, the power distribution must be known before the water density can be found since the density is determined by the heat flux distribution, which in turn depends on the power distribution. What

[16]B. Minkler, private communication.

must be done in this case is to assume some density distribution to start with—say, unity throughout the core. The flux and power distributions are then computed with this water density, and a new density distribution is determined from the computed heat flux. The neutron flux and power distributions are now computed using this new density distribution, and this in turn leads to a third density distribution. The calculations are continued in this way until convergence is obtained—that is, until the density distribution provides a power distribution that gives the same density distribution. Computer programs are available by which such iterative calculations can be carried out automatically. Incidentally, during this preliminary stage in the design of the core, the neutron flux is usually obtained from a multigroup diffusion calculation using only a few groups. To reduce computation costs, multigroup calculations with many groups are normally performed in the final stages of the design.

If the reactor is water cooled, the DNB ratio is next computed as a function of distance along the hottest channel. As pointed out earlier in this section, the minimum DNBR must be equal to some preset limiting value. If the computed DNBR is not equal to this value—it would only be luck if it were at this point—then the parameters of the lattice or the coolant flow are changed, the previous calculations of the core are repeated, and a new DNBR is obtained. These computations are repeated until the desired $(DNBR)_{min}$ is found. This also determines the maximum heat flux, although, as indicated in Fig. 8.13, q''_{max} does not necessarily occur at the point where the DNBR is smallest. If the reactor is not water cooled, q''_{max} is determined solely by the limiting value of the fuel temperature. For either type of reactor, water cooled or not, a preliminary calculated value of q''_{max} has now been found.

At this point, the overall hot channel factor is obtained using the methods discussed earlier. The average heat flux throughout the core can then be computed from Eq. (8.111):

$$q''_{av} = \frac{q''_{max}}{F}. \tag{8.116}$$

By the simple device of designing the reactor to operate at this average heat flux, the actual value of q''_{max} is reduced below its calculated value by the factor F_E. This provides the reactor with the desired margin of safety.

Once q''_{av} is known, the total heat transfer area can be found from Eq. (8.112). Finally, from the heat transfer and the dimensions of the fuel rods, it is possible to compute the total number of rods required in the reactor. This number will undoubtedly be different from the one used at the start of the design, and therefore many of the foregoing calculations must be repeated until a completely self-consistent reactor is obtained. At this point, detailed multigroup and two-dimensional neutron

diffusion calculations can be undertaken, along with various transient analyses and lifetime-burnup computations, to refine the design.

Example 8.12

The core of a PWR consists of a lattice of fuel rods 12 ft long and 0.5 in. in diameter. The reactor operates at a thermal power of 3,000 MW; it has a maximum calculated heat flux of 539,000 Btu/hr-ft^2 and an overall hot channel factor of 2.80. Calculate (a) the total heat transfer area; (b) the number of fuel rods in the core.

Solution.

1. The average heat flux is given by Eq. (8.112):

$$q''_{av} = \frac{539,000}{2.8} = 193,000 \text{ Btu/hr-ft}^2.$$

With a reactor power of 3,000 MW $= 3,000 \times 3.412 \times 10^6$ Btu/hr, the heat transfer area is

$$A = \frac{3,000 \times 3.412 \times 10^6}{193,000} = 53,000 \text{ ft}^2. \text{ [Ans.]}$$

2. The surface area of each fuel rod is $12 \times (0.5\pi/12) = 0.5\pi$ ft ^{2}. The total number of rods is then

$$n = \frac{53,000}{0.5\pi} = 33,740. \text{ [Ans.]}$$

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